Reverse engineering the solution of a "simple" physics problem: Why learning physics is harder than it looks

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Problem solving is the heart and soul of most college physics and many high school physics courses. The "big idea" is that physics tells you more about a physical situation than you thought you knew — and you can quantify it if you use fundamental physical principles expressed in mathematical form. Often, the results of your problem solving can lead you to understand and rethink your intuitions about the physical world in new and more productive ways. As a result, physics is a great place (some of us would claim the best place) to learn how to use mathematics effectively in science.

As physics teachers, we often stress the importance of problem solving in learning physics. Unfortunately, many of our students appear to find problem solving very difficult. Sometimes they generate ridiculous answers and seem satisfied with them. Sometimes they can do the calculations but not interpret the implications of the results. Sometimes, despite apparent success in problem solving, they seem to have a poor understanding of the physics that went into the problems.[1] We give them explicit instructions on how to solve problems ("draw a picture," "find the right equation," …) but it doesn't seem to help.

We might respond that they need to take more math prerequisite classes, but in the algebra-based physics class at the University of Maryland, almost all of the students have taken calculus and earned an A or a B. Many of them have been successful in classes such as organic chemistry, cellular biology, and genetics. Why do they have so much trouble with the math in an introductory physics class?

As part of a research project to study learning in algebra-based physics,[2] the Physics Education Research Group at the University of Maryland videotaped students working together on physics problems. Analyzing these tapes gives us new insights into the problems they have in using math in the context of physics. One problem is that they have inappropriate expectations as to how to solve problems in physics (some of it learned, perhaps, in math classes). This is discussed elsewhere.[3] A second problem seems to lie with the instructors. As instructors, we may have misconceptions about how people think and learn, and this has important implications about how we interpret what our students are doing.

In this paper, we want to consider one example of students working on a physics problem that showed us in a dramatic fashion that we had failed to understand the work the students needed to do in order to solve an apparently "simple" problem in electrostatics. Our critical misunderstanding was failing to realize the level of complexity that we had built into our own "obvious" knowledge about physics.

"Packing" Knowledge Until You Don't See Its Parts: Compilation

Modern cognitive psychology and neuroscience have documented that much of our everyday functional knowledge is dramatically more complex than we give it credit for. One component of this is *automaticity*. Once we have learned to do something, like tie our shoes or ride a bicycle, it becomes easy and we can do it without thinking. But we usually understand and remember the learning that goes into such tasks and we typically have patience in teaching them to our children.

But we have other knowledge that has component parts that are invisible to us. Once our knowledge reaches that stage it is hard to see why someone might not find the result obvious. Even an apparently simple thing like identifying an object is clearly much more complicated than it appears.[4] When we pick up a cup of coffee, its visual image enters the back of our brain through the retina. Tactile data about the feel and heft of the cup enters in the midbrain, the aroma in the forebrain, and the episodic memories that give us the knowledge of what to do with the coffee, what it will taste like, and what its effect will be on us are stored yet elsewhere. Yet the overall result is our sensation of the object as a single integrated and irreducible "thing."

Some cognitive “illusions” dramatically demonstrate how much unnoticed processing the brain is doing for us. A nice example is given by Ed Adelson and is shown in figure 1. The squares of the checkerboard marked A and B are, in fact, exactly the same color. (If you don’t believe this, make a copy of the page with the figure, cut out the squares, and place them next to each other, or check out Adelson’s webpage on the topic.[5]) Your brain knows enough to realize that if two objects appear to be the same color but one is in shadow, then the one in shadow must be “really” lighter — and that’s how you see it. This particular example appears to be “wired up” very tightly and at a very young age. You can’t see it any other way.

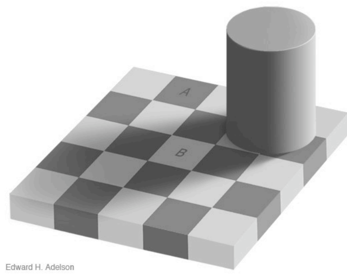


Fig. 1: An example of automatic processing that you cannot unpack. (E. Adelson, with permission)

A second example is more obviously learned. If you look at figure 2, you will find it impossible to look at the words and see any of them (say “cat”) as a series of lines and shapes. You probably not only saw the meaning immediately, you had some visual image associated with the phrase. You have learned to interpret the shapes as letters, to see combinations of letters as words, and to associate the words with particular meanings — objects and actions. Although you can’t undo this easily (looking at it upside down does it for some folks), you know that there was a time when all you could see where lines and shapes.

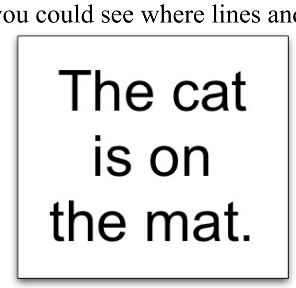


Fig. 2: An example of learned recognition you cannot easily unpack.

This same sort of process occurs as we learn throughout our lives. When professional physicists look at a graph, it is almost impossible for them not to see the y-intercept, the slope at each point, the maxima and minima of the curve. For many students in introductory physics, however, this process is not quick and automatic but takes explicit recall and reasoning.

We refer to the process of binding knowledge tightly so that its parts are inaccessible to the user as *compilation*.[6] The metaphor here is computer code. Once a program written in a high level computer language has been debugged and is stable, it is convenient to convert it into machine language so it doesn’t need to be translated each time it runs. This “executable” is fast, but if you are only given a machine-language executable, it is immensely difficult to back-interpret it to understand what it is actually doing. To understand what is going on in such a program, a computer programmer who wants to re-create it may have to *reverse engineer* it.[7]

Once we learn how to do something, it can be difficult to empathize with someone who does not know how to do that thing. In particular, this lack of empathy may lead physics teachers to forget what it is like to actually learn physics—and, therefore, to not understand how their students are unable to solve “simple” physics problems. Some physics teachers may think, “If it takes a student an hour to solve a problem to which I can just write down the answer, then that student does not know enough physics—and she is wasting her time spending that long on such a ‘simple’ problem.” In this paper we want to demonstrate that this is not the case. To do this, we reverse engineer what solving a “simple” physics problem really entails. We analyze a group of students’ solution to this simple problem and show that, while the students take much longer than the typical teacher to solve this problem, their solution involves many activities that can be seen as part of compiling their physics knowledge and that are appropriate for students at their stage of knowledge.

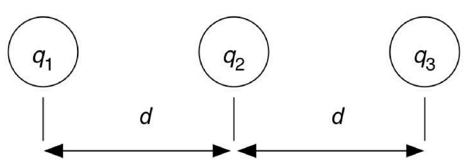
The Setting for this Study.

The setting for this study is an algebra-based physics course at the University of Maryland that was non-traditional in many aspects. The most important non-traditional aspect of this course was its shift in emphasis away from a focus on answers to the idea of understanding principles, process, and concepts. This emphasis was expressed through explicit discussion with the students and through reforms in lecture, recitation, laboratory and homework.[8]

Students were given about five homework problems per week and were told that we expected them to spend about an hour on each problem, even when working together in groups. Many resisted this at first, some not believing it (and trying to do the homework in the fifteen minutes before it was due), some not knowing what work to do for an hour. We gave no exercises and our problems often contained qualitative questions, estimations, and essays. We required that they give explanations in words for each problem in order to receive full credit on the grading. Each week, one problem (not identified for the students beforehand) was graded carefully with written feedback (grade of 0-5). The others received only an indication of whether it was right or not (grade of 0-2). Detailed solutions were posted on the class website. We set up a workroom, the *Course Center*, where students could gather work on their physics homework. The center was staffed approximately twenty hours per week by a teaching assistant (TA) or the course instructor. The staff members assisted the students with their homework, but did not explicitly solve any of the students' homework problems. Often, instead of answering students" questions, TAs directed them to other students working on the problem in the center and they were encouraged to form workgroups.[9] By the end of the first semester, most of the students were working in groups and spending significant amounts of time on their homework (4-6 hours/week).

An Example: the Three-Charge Problem

We examine a videotaped episode in which a group of students attempt to solve an electrostatics problem that we refer to as the *Three-Charge Problem* (figure 3). This episode occurred in the second week of the second semester of the two-semester sequence and shows three female students (pseudonyms, Alisa, Bonnie, and Darlene) working together. All of the students had taken the transformed course in the first semester and were familiar with the idea that a single problem might take a long time to solve and that qualitative and quantitative considerations might both be needed.



In the figure above three charged particles lie on a straight line and are separated by distances d. Charges q_1 and q_2 are held fixed.
Charge q_3 is free to move but happens to be in equilibrium (no net electrostatic force acts on it).

If charge q_2 has the value Q, what value must the charge q_1 have?

Fig. 3: The "three charge" problem.

How instructors solve this problem:

We have asked this problem of numerous physics instructors. Some answer immediately. Others have to think for a few seconds. Occasionally an instructor will give a quick answer and get it wrong at first — but it rarely takes anyone more than a minute to work their way through to a correct answer. A typical instructor's solution might be: "Well, charge 3 is twice as far away from charge 1 as it is from charge 2. So if the forces balance, charge one has to be $-4Q$, opposite sign to balance Q and four times as big because Coulomb's law says the force falls like the square of the distance."

What instructors want students to do

Although most instructors can do this kind of a calculation in their heads, we, as instructors, expected the students to go through a bit more math. The problem states there is "no net electrostatic force" acting on charge q_3. This implies that the sum of all the forces acting on q_3 is equal to zero, which is written formally in symbols as

$$\vec{F}_{q_2 \to q_3} + \vec{F}_{q_1 \to q_3} = 0 \cdot \qquad (1)$$

(We will see below that this form of this equation is deceptively simple and hides a great deal of conceptual information.) Using Coulomb's Law to write the forces in equation (1) yields:

$$\frac{kQq_3}{d^2}\hat{i} + \frac{kq_1q_3}{(2d)^2}\hat{i} = 0 \quad , \qquad (2)$$

where we have set $q_2 = Q$ and written (for simplicity as we did in the class) $k=1/4\pi\varepsilon_0$. Finally, we bring the second term to the right side of the equation and cancel similar terms, which results in the answer: $q_1 = -4Q$.

What the students did

An interesting aspect about the students' problem solving approach is that it takes so long compared to a typical instructor's solution. The students work for nearly 60 minutes before arriving at a solution— almost two orders of magnitude longer than the typical teacher! Is this a problem? In our analysis of our students' approach in solving this and in other problems, two factors seemed critical in understanding what the students were doing.

First, we observe that students tend to solve problems by working in locally coherent activities in which they use only a limited set of the knowledge that they could in principle bring to bear on the problem. We refer to each of these activities as an *epistemic* or *knowledge-building game*. Each game has allowed moves, a starting point, a goal or ending point, and a *form* or visible result. Most important, while students are playing one game, they ignore moves that they consider as not pertinent, thereby excluding much relevant knowledge. We see an example of this in our episode. (These are discussed in detail in ref. 3.)

Second, we note that much of the knowledge that students are using are not integrated; results that would be considered trivially identical by an instructor are treated as distinct and unrelated. They have not yet compiled these distinct knowledge elements the way experts have.

An example of a knowledge-building game many of us have seen when interacting with students is *Recursive Plug-and-Chug.* The student's goal is to calculate a numerical answer. The opening move is to identify the target variable to be calculated. The next move is to find an equation containing that variable. Then, check if the rest of the variables in the equation are known. If so, calculate the resulting quantity. If not, find another target variable in the equation and repeat the process. Note the absence of developing a story about the problem, evaluating the relevance of the equation, or making sense of the answer. The absence of these sense-making moves can produce strikingly inappropriate and even bizarre results. The output form of this game is a string of equations — one that might look identical to that produced by a student playing a more productive game, such as *Making Meaning with Mathematics*.

E-Game	***Description***
Physical Mechanism	using common sense reasoning; "telling the story" of the problem
Pictorial Analysis	using the e-form of a free-body diagram to "nail down" the relevant forces
Mapping Mathematics to Meaning	interpreting current state of mathematical knowledge in terms of the physical elements of the problem
Mapping Meaning to Mathematics	connecting the physical result to using formal knowledge

Table 1: Knowledge games played by the students

Knowledge-building games the students used

In the example we are considering, we can identify five different knowledge-building games that this group of students played to solve the three-charge problem. (See table 1.)

Physical mechanism: Understanding the physical situation. The students start this problem by attempting to understand the physical situation articulated in the problem statement. Their reasoning is based on intuitive knowledge about and experience with physical phenomena rather than on formal physics principles.

Darlene: *I'm thinking that the charge q_1 must have it's...negative Q.*

Alisa: We thought [q_1] would be twice as much, because it can't repel q_2, because they're fixed. But, it's repelling in such a way that it's keeping q_3 there.

Bonnie: Yeah. It has to—

Darlene: Wait, say that.

Alisa: Like— q_2 is— q_2 is pushing this way, or attracting—whichever. There's a certain force between two Q, or q_2 that's attracting.

Darlene: q_3.

Alisa: But at the same time you have q_1 repelling q_3.

Darlene initiates this exchange with a possible solution to this problem: the charge on q_1 is "negative Q." Although this is wrong, it has a good piece of physics: the charge on q_1 must have the opposite sign to the charge on q_2 if the forces they exert on q_3 are to balance. Rather than simply accept or reject this suggestion, the students discuss the physical mechanism that acts to keep q_3 from moving: q_2 is attracting and q_1 is repelling q_3, or vice versa. If the students were only attempting to find a solution to this problem, then a discussion about the physical mechanism seems unnecessary—they would only need to assess the correctness of Darlene's assertion. That the students discuss a possible physical mechanism involved in the physical situation is an indication that the students are attempting to develop a conceptual understanding of this problem. Much of the research on quantitative problem solving in physics discusses the importance of conceptual understanding.[10] Although the instructor's solution outlined above does not explicitly contain a description of the physical mechanism underlying the physical situation, it is clearly implicit — compiled into the way the instructor thinks about and approaches the problem. They not only "know the right formula," they know what the formula means and how to use it.

The next exchange indicates, however, that the students' intuitive ideas about the physical situation are not always consistent with the physics principles that an expert would use. The students are still struggling with reconciling the principle of superposition with their everyday ideas.

Darlene: *How is [q_1] repelling when it's got this charge in the middle?*

Alisa: Because it's still acting. Like if it's bigger than q_2 it can still, because they're fixed. This isn't going to move to its equilibrium point. So, it could be being pushed this way.
Darlene: Oh, I see what you're saying.
Alisa: Or, pulled. You know, it could be being pulled more, but it's not moving.
Darlene: Un-huh.

The arrangement of the charges cues Darlene to think that the presence of q_2 somehow hinders or blocks the effect of q_1 on q_3, which does not agree with the superposition principle. Exploring the possibility that q_2 blocks the effect of q_1 on q_3 is not a step in the instructor's solution outlined above — it's something the instructor knows and takes for granted. The exploration is not a dead-end but a step in helping them to develop the intuitive sense of superposition. Alisa's argument is particular interesting here. She uses an incorrect qualitative argument (*overcoming* — "if it's bigger…it could be being pulled more") rather than the correct quantitative one (superposition) that she later shows she knows. Within the context of the Physical Mechanism game, formal arguments are not "legal" moves.

What we see here (and in many other examples) is that when the students are playing a particular knowledge-building game, they tend not to use other knowledge that they have. That knowledge seems to belong to a different game and not be easily accessed here.

Pictorial analysis: Drawing a picture. The students make progress on this problem by attempting to develop a conceptual understanding of the physical situation in terms of their intuitive ideas; however, this is not stable within the group. After their apparent initial agreement in developing a conceptual understanding of the physical situation, and in particular on determining that charges q_1 and q_2 had to have opposite sign, Darlene decides she is not convinced.
Darlene: I think they all have the same charge.
Bonnie: You think they all have the same charge? Then they don't repel each other.
Darlene: Huh?
Bonnie: Then they would all repel each other.
Darlene: That's what I think is happening.
Bonnie: Yeah, but q_3 is fixed. If it was being repelled—
Alisa: No, it's not. q_3 is free to move.
Bonnie: I mean, q_3 is not fixed. That's what I meant.
Darlene: Right.
Bonnie: So, like...
Darlene: So, the force of q_2 is pushing away with is only equal to d.
Bonnie: Yeah, but then...
Darlene: These two aren't moving.
Bonnie: Wouldn't this push it somewhat?
Alisa: Just because they're not moving doesn't mean they're not exerting forces.
Darlene: I know.
Alisa: What do you think?

The TA (Tuminaro) notices the students' failing to communicate clearly and lock down their apparent gains, and suggests that they draw a picture of the physical situation. The students do not use an algorithmic pictorial analysis technique (*e.g.* free-body diagrams), but rely on their intuitive ideas about the situation to generate a picture. The picture helps the students organize their thoughts and agree on the relative sign on each of the charges.
Alisa: So, maybe this is pushing...
Darlene: That's [q_2] repelling and q_1's attracting?
Bonnie: Yeah, it's just that whatever q_2 is, q_1 has to be the opposite. Right?
Alisa: Not necessarily.
Darlene: Yeah.
Bonnie: OK, like what if they were both positive?
Alisa: Well, I guess you're right, they do have to be different, because if they were both positive...
Bonnie: Then, they'd both push the same way.
Alisa: And, this were positive it would go zooming that way.
Darlene: They would both push.
Alisa: And, if this were negative it would go there.
Bonnie: It would go zooming that way.
Alisa: And, if they were negative...
Darlene: It would still—they'd all go that way.
Alisa: It would be the same thing.

The picture enhances the students' ability to reason about this problem, which enables them to agree on a clear intuitive understanding of the physical situation: "whatever [the sign of] q_2, q_1 has to be the opposite."

Mapping mathematics to meaning: Identifying the relevant physics. At this point, the students have not made use of Coulomb's Law—they have relied solely on their intuitive ideas. Yet, their intuitive reasoning helps them understand the physical situation—and, ultimately, realize that Coulomb's Law is essential for this problem.
Bonnie: Yeah. Negative two Q, since it's twice as far away.
Alisa: And, this is negative Q.
Bonnie: Negative two Q.
Darlene: Negative two Q.

Alisa: Are we going to go with that?
Bonnie: I think it makes sense.
Darlene: That makes...
Alisa: Well, I don't know, because when you're covering a distance you're using it in the denominator as the square.
Bonnie: Oh! Is that how it works?
Alisa: And [...inaudible...] makes a difference.
Bonnie: Yeah, you're right.
Tuminaro: So, how do you know that?
All: From the Coulomb's Law.
Bonnie: So, it should actually be negative four q? Or what? Since it has…
Alisa: Cause we were getting into problems in the beginning of the problem with [the non-equilibrium three-charge problem – see figure 4] because I thought that like if you move this a little bit to the right the decrease for this would make up for the increase for this. But, then we decided it didn't. So, that's how I know that I don't think it would just increase it by a factor of two.

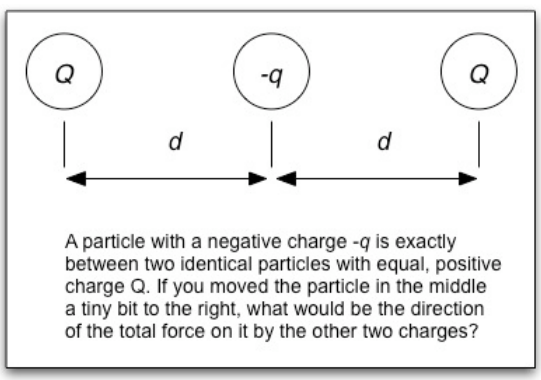


Fig. 4: Another "three charge" problem

The students relied on their intuitive ideas to generate a conceptual understanding of the physical situation—two mutually exclusive influences acting on q_3 that exactly cancel each other. Yet, Alisa, recalling her experience of working on an earlier problem (figure 4) realizes that their intuitive ideas are not enough—they need Coulomb's Law.

Mapping meaning to mathematics: Translating conceptual understanding into mathematical formalism After some false starts and nearly sixty minutes, the students finally solve this problem, integrating their conceptual understanding, which they developed in terms of their intuitive common-sense ideas, with formal application of Coulomb's Law.

Tuminaro: What did you do there?
Alisa: What did I do there?
Tuminaro: Yeah, can I ask?
Alisa: All right, so because this isn't moving, the two forces that are acting on it are equal: the push and the pull.

Alisa reiterates the group's conceptual understanding of the physical situation: two mutually exclusive influences exactly canceling yielding no result.

Next, Alisa writes down the form of the two forces in terms of Coulomb's Law:

Alisa: So, the F—I don't know if this is the right F symbol—but, the F q_2 on q_3 is equal to this [see eq. 3]. And, then the F q_1 on q_3 is equal to this [see eq. 4], because the distance is twice as much, so it would be four d squared instead of d squared.

$$F_{q2\rightarrow q3} = \frac{kQq_3}{d^2} \tag{3}$$

$$F_{q1\rightarrow q3} = \frac{kxQq_3}{4d^2} \tag{4}$$

Alisa: And, then I used x Q like or you can even do—yeah—x Q for the charge on q_1, because we know in some way it's going to be related to Q like the big Q we just got to find the factor that relates to that…Then, I set them equal to each other…

Alisa uses Coulomb's Law to write the form of the two forces, but she does not formally invoke the other physics principle outlined in the ideal solution: Newton's 2nd Law. Rather, Alisa relies on her conceptual understanding of the physical situation to write that the forces must be equal—not on a formal application of Newton's 2nd Law. This feature of her solution is not insignificant; it lends additional evidence that Alisa is making sense of this problem, rather than simply following some problem-solving algorithm.

After setting up the equation, Alisa is only left with an algebra problem, which she has little trouble solving:

Alisa: ... and I crossed out like the q_2 and the k and the d squared and that gave me Q equals x Q over four. And, then x Q equals four Q, so x would have to be equal to four. That's how you know it's four Q.

The other students then evaluate the plausibility and validity of Alisa's recited solution—yet another indication that these students are making sense of this problem.

Bonnie: Well, shouldn't it be—well equal and opposite, but...
Alisa: Yeah, you could stick the negative.
Bonnie: Yeah.

Darlene: I didn't use Coulomb's equation, I just—but it was similar to that.
Bonnie: That's a good way of proving it.
Darlene: Uh-huh.
Bonnie: Good explanation.
Alisa: Can I have my A now?

Alisa's final question is meant in jest ("Can I have my A now?"), but shows that she realizes that she has understood and solved this problem in an expert-type manner.

Doing good work

The students' solution, while it took much longer than the average teacher would take to generate one, has many components of an expert-like solution. However, they show that much of the knowledge they call on is not yet compiled in an expert fashion. The students have to recall and construct the background knowledge needed to make sense of the problem a step at a time: a long but likely necessary step in creating their own knowledge compilations.

Even though it takes them longer than the typical expert, there are two clues that show that the students are working appropriately — in an expert-like manner: they rely heavily on their conceptual understanding and they themselves choose their own path.

First, the students solve this problem by using their intuitive understanding of the physical situation—two mutually exclusive influences exactly canceling each other—and formal application of Coulomb's Law. At no point during the entire problem-solving episode do they use or make explicit reference to Newton's 2nd Law, even though it is the relevant physics principle for why q_3 remains in equilibrium. This is not a negative. It shows that the students are using their own understanding of the physical situation to generate a solution (an expert-like characteristic), rather than doggedly applying a formal physical principle (a novice-like characteristic).

Second, they generate their own problem-solving path. They do not defer to the TA and let him direct what they should do next. The language that the students use is an indication that they do not defer to the TA—even though the students follow the TA's suggestion to draw a picture. The students use phrases like "we thought" or "I decided," and never use phrases like "the TA said" or "the book says," which offers linguistic evidence that the students are in control of the solution path—and not the TA.

Finally, from the details of the full transcript,[11] it is clear that the students are recalling and working through many of the items of basic knowledge that are new to them (only learned last semester) and that they are still reconciling with their intuitive knowledge. A list of some of the physics knowledge the students call on explicitly is given in table 2.

E-Game	*Physics Knowledge Needed*
Physical Mechanism	• Like charges repel, unlike attract • Attractions and repulsions are forces • Forces can add and cancel (one does not "win"; one is not "blocked") • "Equilibrium" corresponds to balanced, opposing forces (not a single strong "holding" force) • Electric force both increases with charge and decreases with distance from charge • Objects respond to the forces they feel (not those they exert) • Charges may be of indeterminate sign and still exert balancing forces on the test charge • "Fixed" objects don't give visible indication of forces acting on them; "free" ones do
Pictorial Analysis	• Only forces on the test charge require analysis • Each other charge exerts one force on test charge • Each force may be represented by a vector • "Equilibrium" corresponds to opposing vectors • Vertical and horizontal dimensions are separable • One dimension is sufficient for analysis
Mapping Mathematics to Meaning	• Electric force both increases with charge and decreases with distance from charge • Electric force decreases with the square of the distance
Mapping Meaning to Mathematics	• Charges of indeterminate sign are appropriately represented by symbols of indeterminate sign • Coefficients may relate similar quantities • Balanced forces correspond to algebraically equal Coulomb's-Law expressions

Table 2: Some of the knowledge required by the student in each game

Recognition vs. formal manipulation

In physics, especially at the college level, we tend to focus our attention on formal manipulation. We often don't realize how much of an expert's success is based on a more fundamental cognitive ability: recognition. There is good evidence from both cognitive and neuroscience that we possess a variety of mental abilities.[12] The ability to handle language and formal reasoning is analogous to serial processing in computer technology — sequential and reasonably slow. The ability to recognize faces and places is analogous to parallel processing — all happening at once and very quickly. When we identify a coffee cup or recognize a friend we don't go through a formal checklist of properties, we just recognize them. Many physicists have had the experience of going through a tedious calculation, making mathematical manipulation after manipulation, and then reaching a particular point and saying: "Oh! Now I get it." Either the rest of the calculation now becomes trivial or the previous steps now are obvious rather than formal.

The sort of thing we mean is easily demonstrated in more daily activities. The anagram puzzle "Jumble" is familiar to many and appears in hundreds of newspapers. (See figure 5.) The task is to rearrange each of four strings of letters into words. The circled letters in each of the four words are combined and rearranged to provide the answer to the phrase clued by the drawing. Before going on to the next paragraph, look at figure 5 and see if any of the words just spring to mind.

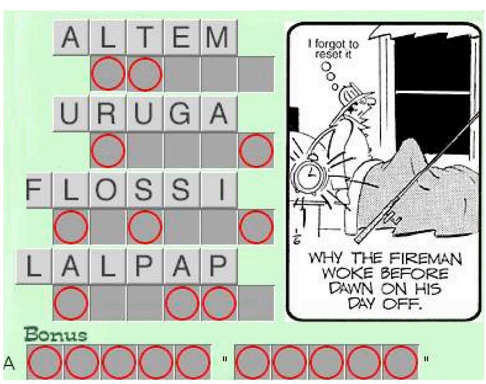


Fig. 5: A puzzle illustrating two kinds of thinking (©Jumble, permission applied for)

Many people immediately recognize either the first or the third word.[13] With these, the bonus phrase ("false alarm") is reasonably obvious from the context. After getting the fourth word (pretty straightforward since there are only three distinct letters), the second word can be approached. The bonus phrase tells us that the two circled letters in the second word must be "a" and "r". This produces the two options: "a __ __ __ r" or "r __ __ __ a" with the letters "u", "u", and "g" to be distributed in the three interior spaces. There are only six possibilities: running through them has one recognizable word: "augur."

Note the two mental processes we've described. In the first case (see note 13) most people simply look at the scrambled letters and know the answer — direct recognition. In the second case ("augur"), most people have to go through a formal algorithm — checking out all of a limited set of possibilities. But in the end, even the second case relies on our recognizing the word "augur." It's a fairly uncommon word and many people don't know it. If the recognition process is missing, the formal manipulations won't help.

Our handling of a physics problem has much in common with the way we think about the Jumble puzzle. We do formal manipulation until we reach a point where we recognize the result as making sense. But when we have a lot of compiled knowledge, our intuitive recognition skills become much stronger. When we focus on teaching our students formal manipulation skills, we are tacitly assuming that the recognition skills will grow naturally. But if we are unaware of how compiled our knowledge is, we may not appreciate the work students need to do to compile their new knowledge and build their intuitive recognition skills.

In this paper, we reverse engineered a simply physics problem, comparing the way instructors who already have lots of compiled knowledge solve it to the way a group of novice students who are still working on compiling their knowledge solve it. An immediately obvious difference between the two solutions is that the students' solution took much longer to generate—nearly two orders of magnitude longer. Our analysis shows two things. First, even simple physics problems are difficult for novices. There are many conceptual and technical subtleties to physics problems that experts tend to forget about because they are so familiar with these subtleties that they don't notice them. Second, what we may at first judge to be poor student problem-solving behavior may actually be very good behavior. After careful analysis of the students' solution in the three-charge problem, we see that the students' solution shares many aspects with the expert-like solution. In addition, the students are doing work towards the consolidation and reconciliation of new knowledge, which is just what they need to be doing at their stage of learning. To do a better job helping our students, we need to better understand both what they know and the hidden components of our own knowledge. Only then, can we effectively "reverse-engineer" what we know to figure out what our students have to go through to build expert problem-solving skills.

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[4] O. Sacks, *The Man Who Mistook His Wife for a Hat* (Touchstone, 1998).
[5] http://web.mit.edu/persci/people/adelson/checkershadow_proof.html
[6] This is often referred to in the cognitive and neuroscience literatures as *binding*. They often do not, however, distinguish between permanent binding that is irreducible for the user and dynamic binding that is temporary and situational. We introduce *compilation* to focus on permanent bindings that look trivial and obvious to the user.
[7] "Reverse engineering is the process of taking something (a device, an electrical component, a software program, etc.) apart and analyzing its workings in detail, usually with the intention to construct a new device or program that does the same thing without actually copying anything from the original." http://en.wikipedia.org/wiki/Reverse_engineering
[8] See ref. 3 for more details.
[9] Students in the Course Center typically spontaneously formed workgroups with other students who happened to be there. When a group worked well, they would sometimes arrange a regular meeting time for subsequent collaborations.
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[11] The full transcript is available online at http://www.physics.umd.edu/perg/dissertations/Tuminaro/3Qtranscript.pdf.
[12] G. Fauconnier and M. Turner, *The Way We Think: Conceptual Blending and the Mind's Hidden Complexities* (Basic Books, 2003), Chapter 1.
[13] "Metal" and "fossil". Each of the anagrams for these words contains a significant block of the letters of a familiar word in the correct ordering. The third ("appall") may be a bit harder.