Optical Pumping

and the Hyperfine Structure of Rubidium

Goal: Investigate hyperfine and Zeeman interactions in rubidium.

Introduction

Atomic Angular Momentum

There are a few contributions to the angular momentum of an atom.

L: Orbital angular momentum

This angular momentum is due to the motion of the electrons around the nucleus. The capital L indicates the total angular momentum due to all electrons in the atom. We are interested in the lowest-energy states, in which L = 0.

S: Electron spin angular momentum

Spin is an inherent property of electrons. The capital S indicates the total spin angular momentum due to all electrons in the atom. The spin of a single electron is s = 1/2. In atoms like rubidium with a single unpaired electron, the lowest energy states have S = 1/2.

J: Total electron angular momentum

This is the total electron angular momentum due to orbital angular momentum and spin. If L = 0 and S = 1/2, J = L + S = 1/2.

I: Nuclear spin angular momentum

The protons and neutrons in the nucleus have spin too. Nuclear physics shows that rubidium-85 and rubidium-87 have different values of I. You will determine I for these isotopes of rubidium.

F: Total atomic angular momentum

The total atomic angular momentum combines J and I. The rule is that

$$|J - I| \leq F \leq J + I. \quad (1)$$

Since J = 1/2 and I > 1/2, F is either I – 1/2 or I + 1/2.

Z Projection of Angular Momentum

The total angular momentum F is a vector. If a magnetic field is applied in the z direction, and we measure the z component of F, we find that it is quantized: The only possible values are proportional to a number m_F, whose values are -F, -F + 1, … F – 1, F. For example, if F = 2, the possible values of m_F are -2, -1, 0, 1, and 2.

The same rules apply to the z components of L, S, J, and I. For example, with S = 1/2, the possible values of m_S are -1/2 and 1/2.

Dimensionless vs. Dimensional

TeachSpin uses non-boldfaced symbols L, J, I, and F to represent dimensionless numbers. These are all integers or half integers. These dimensionless numbers are related, but not identical, to the physical values that can be measured. The physical value of orbital angular momentum is $\hbar\sqrt{L(L+1)}$, the physical value of total electron angular momentum is $\hbar\sqrt{J(J+1)}$, the physical value of nuclear spin angular momentum is $\hbar\sqrt{I(I+1)}$, and the physical value of total atomic angular momentum is $\hbar\sqrt{F(F+1)}$.

Similarly, m_L, m_J, m_I, and m_F are dimensionless numbers proportional to the z projections $L_z = m_L\hbar$, $J_z = m_J\hbar$, $I_z = m_I\hbar$, and $F_z = m_F\hbar$.

TeachSpin uses boldfaced symbols to represent (dimensional) vector operators. An operator is something that acts on a quantum state. If the state is an **eigenstate** of the operator, the action of the operator on the state is to multiply it by the **eigenvalue**. The eigenvalue is the physical value that would be measured. For example, a state with specific values of J and m_J can be written $|J,m_J\rangle$. Then, the operators $\mathbf{J^2}$ and J_z can act on the state to generate the values that would be measured:

$$\mathbf{J^2}|J,m_J\rangle = \hbar^2 J(J+1)|J,m_J\rangle, \quad (2)$$

and

$$J_z|J,m_J\rangle = \hbar m_J|J,m_J\rangle. \quad (3)$$

Similar equations apply to $\mathbf{L^2}$, $\mathbf{I^2}$, $\mathbf{F^2}$, L_z, I_z, and F_z.

Magnetic Moment

Image an electron with mass m_e and charge -e traveling with speed v in a circle with radius r. The angular momentum is $L = m_e vr$. Since current is charge/time, and the time required to orbit the circle is $2\pi r/v$, the current $i = -ev/2\pi r$. We know that current-carrying loops interact with magnetic fields. You may recall

that magnetic moment μ is current times the area of the loop. So the magnetic moment of the electron, due to angular momentum L, is

$$\mu_L = i(\pi r^2) = -ev/2\pi r\,(\pi r^2) = -evr/2. \quad (4)$$

The gyromagnetic ratio is defined as

$$\gamma_L = \mu_L/L = (-evr/2)/m_e vr = -e/2m_e. \quad (5)$$

We're interested in L = 0 (so $\mu_L = 0$), so we need to consider magnetic moment due to spin. The gyromagnetic ratio due to spin has an extra factor called g (which is about 2 for electrons):

$$\boldsymbol{\mu_J} = \gamma_J \boldsymbol{J} = g_J \frac{-e}{2m_e}\boldsymbol{J}. \quad (6)$$

In terms of the Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$,

$$\boldsymbol{\mu_J} = -g_J \mu_B \frac{\boldsymbol{J}}{\hbar}. \quad (7)$$

The nuclear spin I also has magnetic moment, which TeachSpin writes in terms of electron mass (implicit in the Bohr magneton):

$$\boldsymbol{\mu_I} = -g_I \mu_B \frac{\boldsymbol{I}}{\hbar}. \quad (8)$$

Since electron mass is in the denominator in Eq. (6), the much larger proton mass means that $|g_I| << |g_J|$, in the (unconventional) TeachSpin convention.

Hyperfine Interaction

The electron angular momentum **J** interacts with the nuclear spin **I**. The energy of this interaction is proportional to the dot product of **I** and **J**:

$$H = hA \frac{\boldsymbol{I}}{\hbar} \cdot \frac{\boldsymbol{J}}{\hbar}, \quad (9)$$

where A is a constant, and H is the energy operator (called the Hamiltonian).

To simplify $\boldsymbol{I} \cdot \boldsymbol{J}$, we use **F** = **I** + **J**, which means $\boldsymbol{F^2} = \boldsymbol{F} \cdot \boldsymbol{F} = \boldsymbol{I^2} + 2\boldsymbol{I} \cdot \boldsymbol{J} + \boldsymbol{J^2}$, so

$$\boldsymbol{I} \cdot \boldsymbol{J} = \frac{1}{2}\left(\boldsymbol{F^2} - \boldsymbol{I^2} - \boldsymbol{J^2}\right). \quad (10)$$

Combining Eqs. (9) and (10),

$$H = \frac{hA}{2\hbar^2}(\boldsymbol{F^2} - \boldsymbol{I^2} - \boldsymbol{J^2}). \quad (11)$$

When this operator acts on a state |I,J,F⟩ with specific values of I, J, and F, we can use Eq. (2) and equivalent expressions for $\mathbf{F}^2$ and $\mathbf{I}^2$:

$$H|I,J,F\rangle = \frac{hA}{2}[F(F+1) - I(I+1) - J(J+1)]|I,J,F\rangle. \quad (12)$$

Next we use the fact that J = 1/2, so F = I + 1/2 or F = I – 1/2, according to Eq. (1). Do the arithmetic to show that

$$H|I,J=\tfrac{1}{2},F=I+\tfrac{1}{2}\rangle=\frac{hA}{2}I|I,J=\tfrac{1}{2},F=I+\tfrac{1}{2}\rangle \tag{13}$$

and

$$H\left|I,J=\tfrac{1}{2},F=I-\tfrac{1}{2}\right\rangle=-\frac{hA}{2}(I+1)\left|I,J=\tfrac{1}{2},F=I-\tfrac{1}{2}\right\rangle. \tag{14}$$

Equation (13) says that the state |I,J=1/2,F=I+1/2⟩ has energy hAI/2. Equation (14) says that the state with lower F, |I,J=1/2,F=I-1/2⟩, has (lower) energy -hA(I+1)/2. So the energy difference

$$\Delta E=\frac{hA(2I+1)}{2}. \tag{15}$$

This is the hyperfine splitting of the ground state. The energy depends on whether electron spin and nuclear spin are parallel or antiparallel. However, the energy does not depend on the total z projection of angular momentum, m_F.

A photon with energy ΔE and frequency Δν = ΔE/h stimulates the transition between the two hyperfine states. Equation (15) lets us solve for A in terms of this photon frequency as

$$A=2\Delta\nu/(2I+1). \tag{16}$$

Then, Eq. (13) says that the state |I,J=1/2,F=I+1/2⟩ has energy

$$E(F=I+1/2) = hI\Delta\nu/(2I+1) \tag{17}$$

and Eq. (14) says that the state I,J=1/2,F=I-1/2⟩ has energy

$$E(F=I-1/2) = -h(I+1)\Delta\nu/(2I+1). \tag{18}$$

It is convenient to combine Eqs. (11) and (16) into

$$H=\frac{h\Delta\nu}{\hbar^2(2I+1)}(\boldsymbol{F^2}-\boldsymbol{I^2}-\boldsymbol{J^2}). \tag{19}$$

Zeeman Interaction

When we apply a magnetic field **B** in the z direction, the energy acquires two additional contributions: the interaction of the magnetic field with electron angular momentum, and the interaction of the magnetic field with nuclear spin. The energy of a magnetic moment in a magnetic field is $-\boldsymbol{\mu}\cdot\boldsymbol{B}$, so using Eqs. (7) and (8) for $\boldsymbol{\mu}$ makes the total energy

$$H=\frac{h\Delta\nu}{\hbar^2(2I+1)}(\boldsymbol{F^2}-\boldsymbol{I^2}-\boldsymbol{J^2})+g_J\mu_B\frac{\boldsymbol{J}}{\hbar}\cdot\boldsymbol{B}+g_I\mu_B\frac{\boldsymbol{I}}{\hbar}\cdot\boldsymbol{B}. \tag{20}$$

Since **B** is in the z direction, we get the z projections of **J** and **I**:

$$H=\frac{h\Delta\nu}{\hbar^2(2I+1)}(\boldsymbol{F^2}-\boldsymbol{I^2}-\boldsymbol{J^2})+g_J\mu_B\frac{J_z}{\hbar}\cdot B+g_I\mu_B\frac{I_z}{\hbar}\cdot B. \tag{21}$$

The magnetic field further splits the hyperfine states according to the z projection m_F. This is called the Zeeman effect. Consider F=I+1/2. Since m_F is allowed the values -F, -F+1, …, F-1, F, there are 2F+1 allowed m_F values, or 2(I+1/2)+1=2I+2 states.

To find the eigenvalues of Eq. (21), we need to know m_J and m_I as well as F, I, and J. In general, this is a complicated problem (https://en.wikipedia.org/wiki/Zeeman_effect#Intermediate_field_for_j_.3D_1.2F2). However, we can quickly solve a couple special cases. Consider the state with the greatest possible m_F, $|F=I+1/2,m_F=I+1/2\rangle$. Since $m_F = m_I + m_J$, and the greatest possible value of m_I is I, and the greatest possible value of m_J is J = 1/2, we must have $m_I = I$ and $m_J = 1/2$. Now we can let Eq. (21) act on the state $|F=I+1/2,m_F=I+1/2;m_I=I;J=1/2,m_J=1/2\rangle$, using Eqs. (17) and (3) to find the energy eigenvalue:

$$E(\text{greatest } m_F) = hI\Delta\nu/(2I+1) + g_J\mu_B B(1/2) + g_I\mu_B BI. \quad (22)$$

Similarly, when $m_F = -F$ so that $m_I = -I$ and $m_J = -1/2$,

$$E(\text{lowest } m_F) = hI\Delta\nu/(2I+1) - g_J\mu_B B(1/2) - g_I\mu_B BI. \quad (23)$$

So the Zeeman splitting between the highest and lowest m_F states is

$$\delta E = (g_J + 2Ig_I)\mu_B B. \quad (24)$$

Each of the 2I+2 values of m_F corresponds with a different energy. So there are 2I+1 gaps between adjacent energy levels. It can be shown that all these gaps are approximately evenly spaced when B is small. So each energy gap is $\delta E/(2I+1)$. Defining δf as the photon frequency corresponding with the energy gap,

$$\delta f = \frac{1}{2I+1}\frac{\delta E}{h} = \frac{g_j+2Ig_I}{2I+1}\frac{\mu_B B}{h}. \quad (25)$$

Since $\mu_B B/h = 14$ kHz/μT, $g_J = 2$, and g_I is negligible in comparison,

$$\delta f = \frac{28\ kHz}{2I+1}\frac{B}{1\ \mu T}. \quad (26)$$

You can measure photon frequency δf, as a function of the B required to allow transitions among the Zeeman levels. From the slope, you can determine nuclear spin I. Wow!

At higher B, each of the 2I+1 gaps is a little different from the others, so δf splits into 2I+1 different frequencies. Equivalently, for a fixed δf, 2I+1 different B values enable Zeeman transitions. So just by counting the magnetic field values at which you observe a Zeeman transition, you can determine I!

Finally, when B is high and there are many photons, you observe double quantum transitions: Two photons, each of energy $h\delta f$, combine to stimulate a transition of energy $2h\delta f$. This causes m_F to change by 2, over energy gaps of $h\delta f_1$ and $h\delta f_2$. So δf of the double quantum transition occurs halfway between δf_1 and δf_2: When there are many photons, you observe additional frequencies halfway between adjacent frequencies that were present originally.

Optical Pumping

We shine light through a gas of rubidium atoms. Photons carry angular momentum. When photons with a particular circular polarization are absorbed by atoms, the effect is to increase m_F. Eventually all the atoms are in the state of greatest m_F, so no additional photons can be absorbed: the gas becomes transparent. We will observe the intensity of the light passing through the rubidium gas. We say that the atoms were "pumped" into the nonequilibrium state of greatest m_F.

Optical pumping depends on a magnetic field to create energy differences among the different m_F states. So when the field is 0, optical pumping doesn't occur, the atoms absorb more light, and the light intensity at the detector decreases.

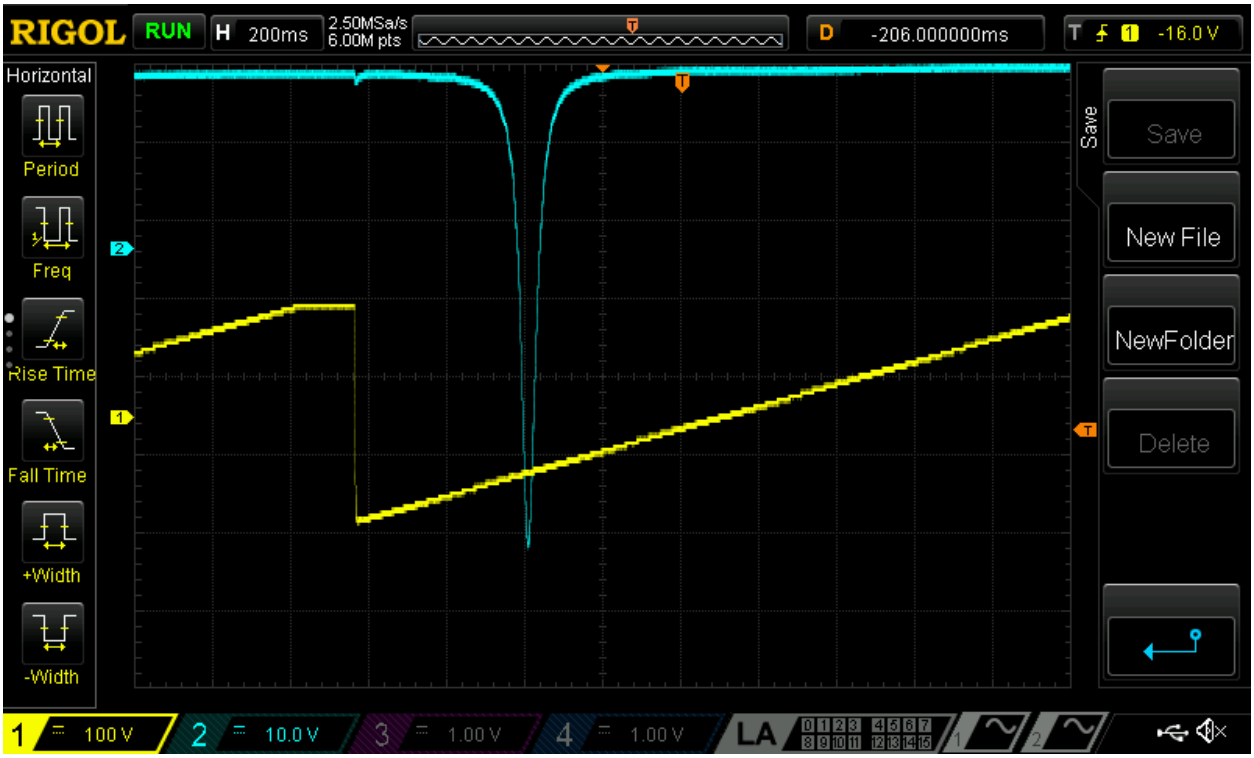
Experiment

The Zero Field Transition

Turn on the optical pumping control box and the oscilloscope. Wait until the temperature stabilizes at 50°C. The oscilloscope is configured to show two voltages as a function of time:

- A sawtooth wave proportional to horizontal magnetic field "sweep" over a range of values. Locate the upper HORIZONTAL MAGNETIC FIELD SWEEP section of the control box. Experiment with adjusting the range, start value, and time of the sweep. For the initial experiments, you'll want the greatest range, the smallest start field, and a time of about 2 seconds.
- A signal related to the light intensity recorded at the detector. Locate the DETECTOR AMPLIFIER section of the control box.
    - We want the arrow of the galvanometer to be within range of the dial. To move the arrow, turn the DC OFFSET dial.
    - We want to motion of the arrow to nearly fill the dial. Turn the GAIN knob to adjust the range of the arrow's motion.

You should see something like this:



We want the dip to be as narrow as possible. This is achieved by minimizing the magnetic field in the vertical direction, and in the direction perpendicular to the length of the apparatus. The apparatus should be aligned with the Earth's magnetic field so there's no horizontal component perpendicular to the apparatus. The vertical component of the Earth's magnetic field is canceled by applying a current through coils. Locate the VERTICAL MAGNETIC FIELD section of the control box. The best value here is usually about 2.7. Make tiny adjustments to this dial to make the dip as narrow as possible.

Basic oscilloscope controls:

- Locate the HORIZONTAL section of the controls. The small, upper knob adjusts the horizontal position of the waveform. The large, lower knob adjusts the horizontal scale (time).
- Locate the VERTICAL section of the controls. If you press CH1 or CH2, the corresponding indicator will be highlighted on the lower right of the display. You can turn the small knob to adjust position and the large knob to adjust scale (voltage).
- To copy the oscilloscope screenshot onto a flash drive, press Storage, then the gray button beside Save, then select the removable disk, then New File.

Hyperfine Transitions

The photons with frequency δf stimulate atomic transitions if they equal the Zeeman spacing given in Eq. (26). So the necessary δf is proportional to the total magnetic field B. Equivalently, the necessary B is proportional to δf. To create photons of frequency δf, we apply a voltage with this frequency to coils.

Press the Source button on the oscilloscope. On the right side of the display, change the Output from OFF to ON by pressing the top gray button. Press the next gray button to access Src1Conf (source 1

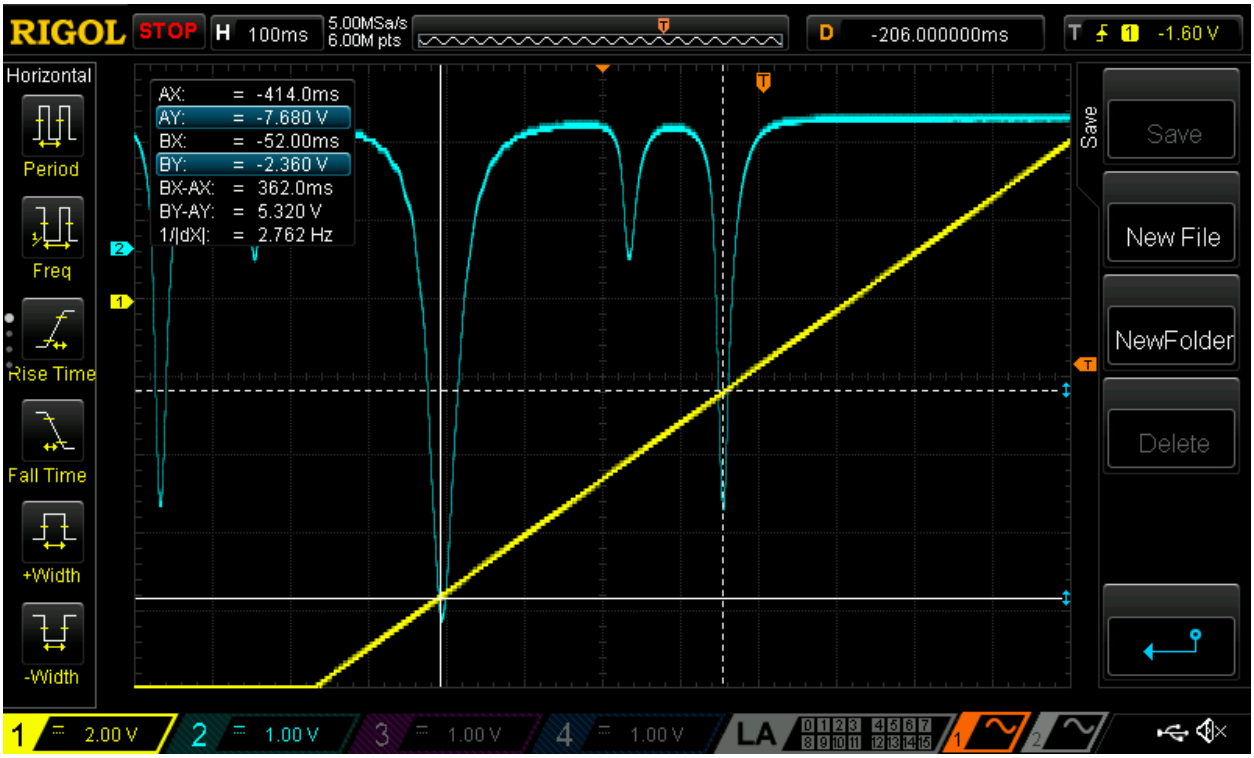
configuration). The Wave should be Sine. The frequency should be at least 30 kHz. To adjust frequency, press the third gray button. Then, there are two ways to change the value: Either turn the upper left knob, or press that knob to bring up a number pad. Turn the knob to select the number, then press the knob to select it. Sometimes the cursor shifts as you press the knob. It may be helpful to squeeze the knob as you press it.

Slowly increase δf, and observe that the side dips move further from the center. You should see something like this:

Notice the symmetry around the central dip. Recall that the central dip represents zero magnetic field. Now consider a nonzero field B. It can point in either direction along the length of the apparatus. So the regions to the right and left of the central dip indicate magnetic fields in opposite directions.

Why are there two side dips on each side? Because there are two isotopes of rubidium in our gas, with different nuclear spins I in Eq. (26). We want to determine these two I values. To do this, we want to plot δf vs. B for both side dips. It's easy to know δf: Read it off the oscilloscope. To determine B at one side dip, we first must place vertical cursors on the central (B=0) dip and the side dip:

Press the Cursor button. Click the gray button and select Manual to turn cursors on. Turn the knob to position one vertical cursor on the central dip. Then press the knob to switch to the other vertical cursor, and place it on the side dip. Press Select to switch to horizontal cursors. Now, observe where the vertical cursors cross the **yellow** line, and place the horizontal cursors on the intersections of the **yellow** line with the vertical cursors:



Now, BY-AY = 5.32 V gives a value proportional to the magnetic field required to create the side dip, the B in Eq. (26). First we need to convert this voltage to the current through the coils. Locate the lower HORIZONTAL MAGNETIC FIELD SWEEP section of the control box. Notice the RECORDER OUTPUT which goes to the oscilloscope (plotted as the yellow line) and the MONITOR (the voltage across a 1 Ω resistor in the coils). You need to determine the constant of proportionality between the RECORDER OUTPUT and the MONITOR. (I find that the RECORDER OUTPUT stops changing when the MONITOR approaches its highest value. So look for a linear relationship in the range where RECORDER OUTPUT is changing.)

Once you're able to convert RECORDER OUTPUT voltage to current through the coils, you can use the approximate coil constant of 60 μT/A from the TeachSpin manual to convert to magnetic field B.

So, determine δf vs. B for both side dips (due to symmetry, you don't have to examine all four side dips, though you may), and use Eq. (26) to determine nuclear spin I for both rubidium isotopes.

High-Field Effects

We would like to see effects of high magnetic field. To do this, connect the HORIZONTAL FIELD cables to the control box. The dial in the HORIZONTAL MAGNETIC FIELD section controls the steady magnetic field (to which the sweep field is added). As you increase B, you need to increase δf to continue to see the side dips. Try to judiciously increase both B and δf to continue to produce one of the side dips. Keep track of which dip you're following! At high enough field, the side dips resolve into multiple dips:

To avoid double-quantum transitions, you will need to reduce the amplitude of the high-frequency (δf) voltage you are applying. The amplitude should be less than 0.5 V. Now, if you count the number of dips you can determine I! As shown at the end of the introduction, the number of magnetic field values is 2I+1. Use this method to determine I for the two isotopes of rubidium.

When you increase the voltage amplitude, you see additional dips (double quantum transitions) at the midpoints of the original dips:


RIGOL
STOP
H 200ms
2.50MSa/s
6.00M pts
D -300.000000ms
T 9.80 V
Horizontal
Period
Freq
Rise Time
Fall Time
+Width
-Width
Save
Save
New File
NewFolder
Delete
1 5.00 V
2 500mV
3 1.00 V
4 1.00 V
LA