QUANTUM ENTANGLEMENT (WOW!)

CAUTION: A 20 mW violet laser is used in this experiment. Never look directly into the beam, including any reflected beams! Be mindful of accidental reflection off watches, jewelry, or other shiny objects! Avoid prolonged exposure of the skin to the beam, or it will begin to burn!

A RIDDLE

Suppose you flip a coin and hide it in your fist. When you open your hand, you see that the coin is heads. You conclude (correctly) that the coin was heads even before you opened your hand; opening your hand didn't change the state of the coin; the observation merely gave you information about the coin. But when you measure the polarization of a photon, do you detect the polarization that the photon had *all along*, or does the measurement fundamentally alter the state of the photon?

We might say that "hidden variables" (quantitative details about the way you flipped the coin and caught the coin) precisely determined the final orientation of the coin. We don't know the values of the hidden variables (that's why they're "hidden"), but in principle they could've been unhidden through sufficiently precise observations. Are there hidden variables in quantum mechanics, or prior to measurement, might a quantum state be fundamentally unknowable?

INTRODUCTION

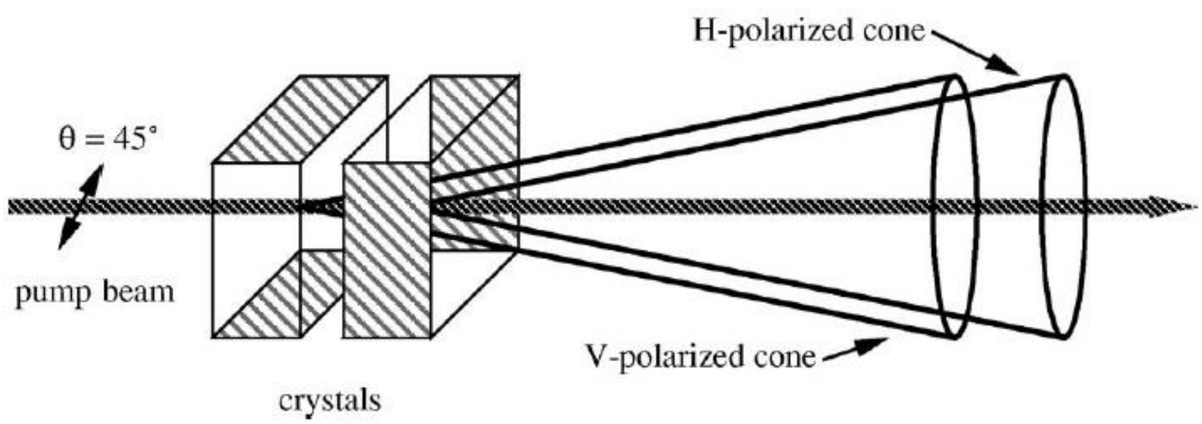

Figure 1. From Dehlinger and Mitchell, Am. J. Phys. **70**, 903-910 (2002). Pair of beta barium borate crystals. The thick arrow represents incoming 405 nm light. The cones represent 810 nm light produced when a 405 nm photon splits within a crystal.

We shine 405 nm violet light on a pair of beta barium borate (BBO) crystals, as shown in Figure 1. One of the BBO crystals is orientated to interact with horizontally polarized violet light, and the other is orientated to interact with vertically polarized violet light. If an incoming violet photon is **horizontally polarized**, it may split into two 810 nm infrared photons (at opposite sides of the cone) with **vertical polarization**. If an incoming violet photon is **vertically polarized**, it may split into two 810 nm infrared photons (at opposite sides of the cone) with **horizontal polarization**. This splitting of photons is called **spontaneous parametric downconversion**. We will set up our apparatus to detect infrared photons at two positions on opposite sides of the cone, as shown in Figure 2.

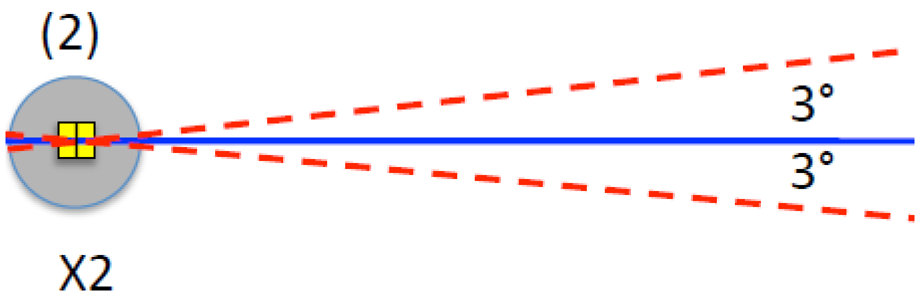

Figure 2. From Galvez (Colgate University). Horizontal "slice" of Figure 1. X2 is the crystal pair. The 810 nm photons travel at a 3° angle to the beam of 405 nm photons.

What happens when the incoming light is polarized at a 45° angle, as shown in the Figure 1? 45° polarization is a superposition of horizontal and vertical polarization. So each incoming photon has an equal probability of interacting with either of the crystals (but not both), and the resulting polarization of the of infrared photons is written

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B\right), \quad (1)$$

where A represents the infrared photon at one side of the cone, and B represents the infrared photon at the other side of the cone. This expression means that the two photons were produced in the same crystal, but we don't know which one, so we don't know if the photons are horizontally or vertically polarized. But if measurement shows one photon to be horizontally polarized, the other one must be horizontally polarized as well. Likewise, if measurement shows one photon to be vertically polarized, the other one must be vertically polarized.

Imagine that we direct the two infrared beams toward polarizers. We want to determine the probability that the photons are transmitted, as a function of the angles of the polarizers. If a polarizer is at angle α relative to the vertical, we can imagine a polarization state $|\alpha>$ as a superposition of $|V>$ and $|H>$:

$$|\alpha> = \sin\alpha|H> + \cos\alpha|V>. \quad (2)$$

You can derive this by simply decomposing $|\alpha>$ into vertical and horizontal components. If you're not familiar with |this notation>, just think of it as a way of representing a vector that may have horizontal and vertical components, equivalent to $\hat{\imath}\sin\alpha + \hat{\jmath}cos\alpha$.

If the polarizer in front of A is at angle α, and the polarizer in front of B is at angle β, the **probability $P(\alpha,\beta)$ that <u>both photons</u> pass through the polarizers** is written

$$P(\alpha,\beta) = \left| \langle\alpha|_A \langle\beta|_B |\psi\rangle \right|^2 . \qquad (3)$$

In your lab report, you will prove that this equals **$\frac{1}{2}\cos^2(\beta-\alpha)$**. Why does this result make sense? Suppose one photon reaches a polarizer slightly before the other. If the first photon goes through a polarizer at angle α, the other photon (for all practical purposes) immediately becomes polarized in the same direction because the two photons are "entangled." The second photon's probability of going through a polarizer at angle β is now given by Malus's law, $\cos^2(\beta-\alpha)$. (Above, the factor of ½ occurs because the first photon might not pass through the first polarizer.)

We will discuss quantum entanglement and Bell's theorem below. First, let's just go through the details of the experiment.

EXPERIMENT

Every step must be completed meticulously! The laser beam is very narrow, and if the equipment is misaligned by even 1 mm, the experiment might not work!

1. Aligning the 405 nm laser

First you need to make sure that the 405 nm laser is at the same height as the BBO crystal pair, as shown in Figure 3.

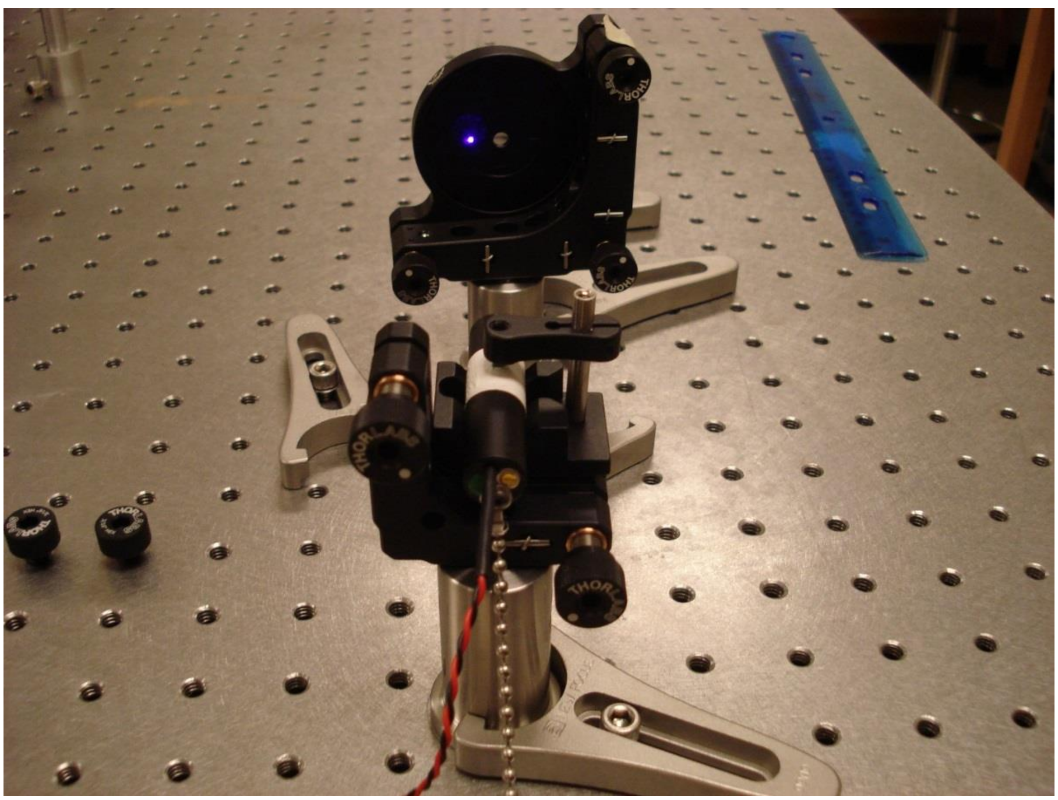

Figure 3. Making sure the 405 nm laser is at the same height as the crystal pair (the small clear circle in the center of the circular black mount).

You'll need to make the beam parallel to the table top, exactly above the row of holes leading to the center of the curved steel plate. To accomplish this, set two irises at exactly the height of the laser. Place an iris very close to the laser (Figure 4), and adjust the height of the iris until the beam goes straight through. Next, tighten a collar against the top of the post holder (Figure 5) holding the iris. This will preserve the height of the iris while allowing you to rotate the post within the post holder. (You will screw the post holders directly into the table, so the post holders won't be able to rotate. To rotate the post within the post holder, loosen the bolt in the post holder. The collar will preserve the height of the post.)

Repeat with the other iris.

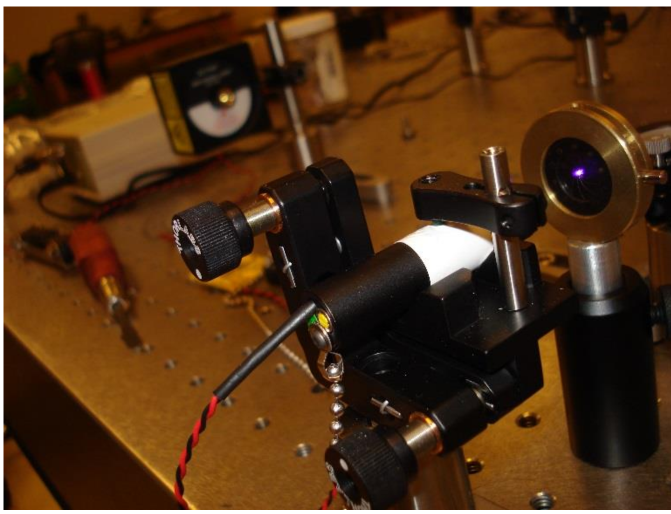

Figure 4. Setting iris to height of laser.

Figure 5. Placing collar to preserve height of iris.

In subsequent photos, the iris posts are held in shorter, less reliable post holders. Please continue to use the post holders shown in Figures 4 and 5.

Screw the post holder for one iris into the bolt hole where the crystal pair will go, 1 m from the steel plate (Figure 6).

Screw the post holder for the other iris into a hole along the same row of holes, near the steel plate (Figure 6).

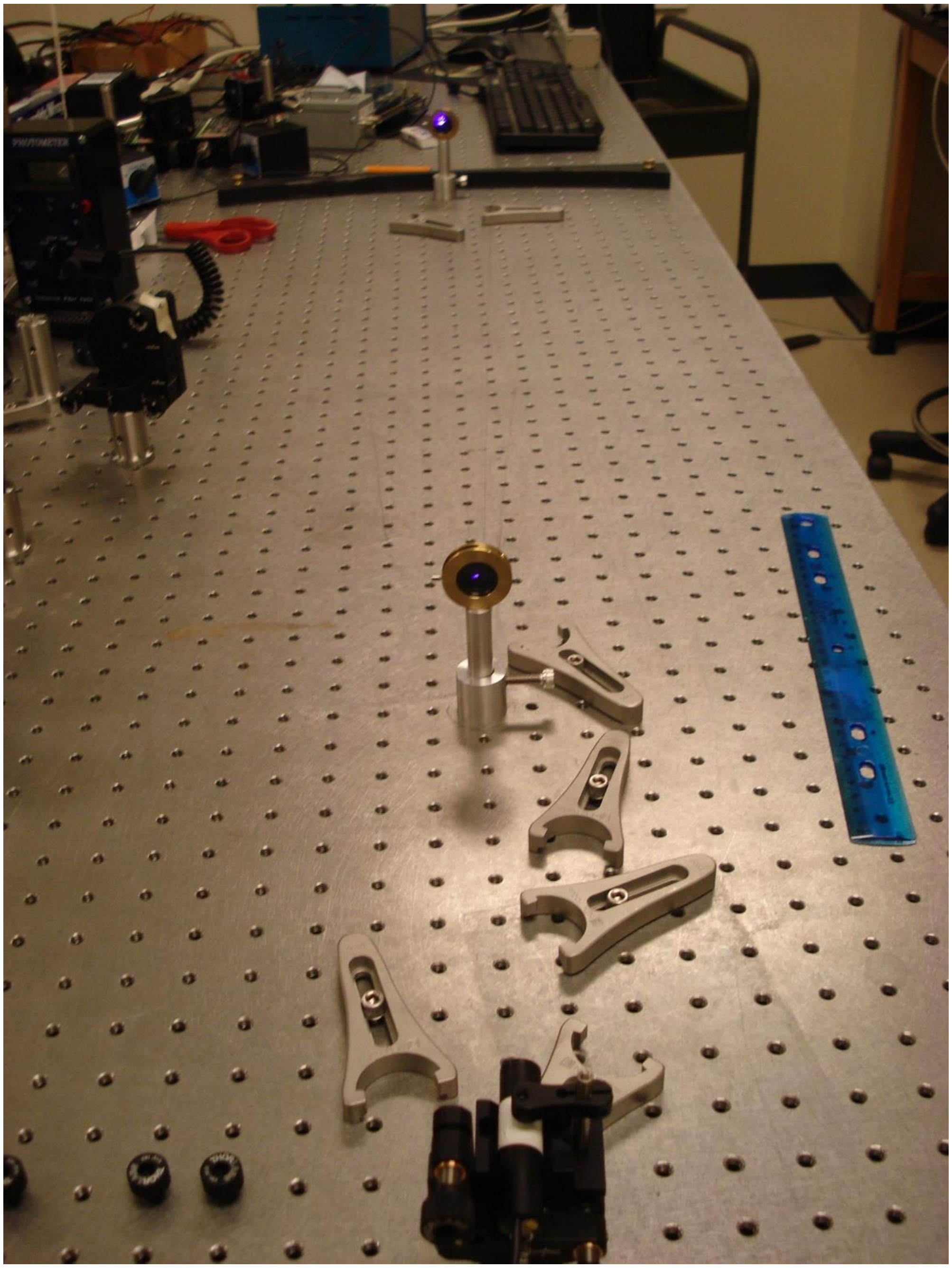

Figure 6. The irises are positioned over the row of holes leading to the center of the steel plate, and the 405 nm laser is aligned and tilted until its beam passes through both irises.

Adjust the position and tilt of the 405 nm laser until the beam goes through both irises. (Our laser produces a bright spot with a “bar” of light extending out in one direction. Ignore the bar; only align the bright spot.) Position the laser as close to the short edge of the table as you can, so it won’t get in the way of the mirrors you’ll soon set up. Clamp the laser in place.

2. Alignment of HeNe laser (along paths to be followed by invisible infrared photons)

Since the infrared beams are invisible, we will direct a red HeNe laser beam along the exact paths that the infrared beams will follow, to align the equipment the will detect the infrared photons.

Make sure the HeNe laser is at the same height as the 405 nm laser and irises (Figure 7).

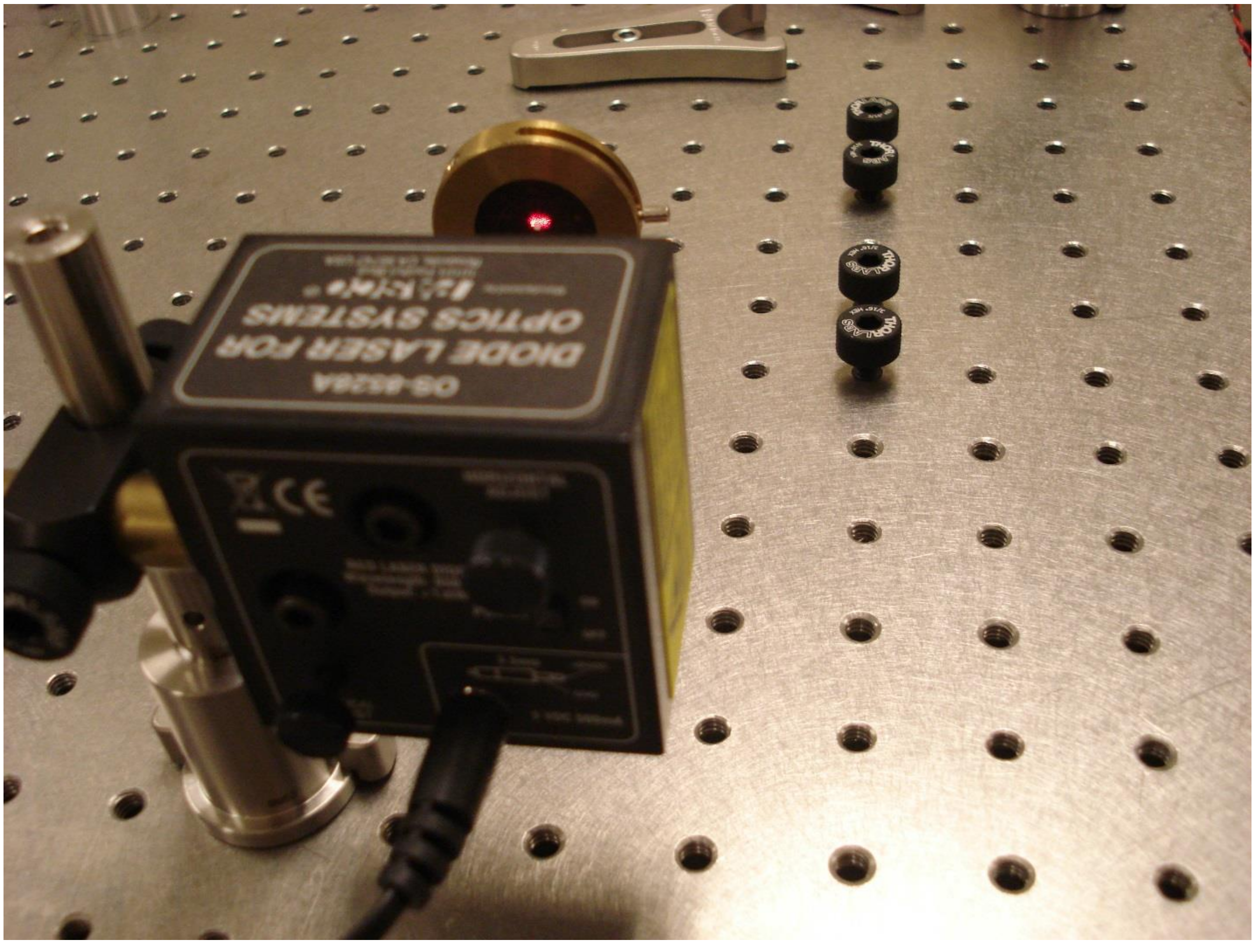

Figure 7. Place an iris near the HeNe laser and adjust the laser's height if necessary.

Adjust the vertical tilt of the HeNe laser if necessary so that its beam is perpendicular to the table; it must go through two irises at the correct height (Figure 8).

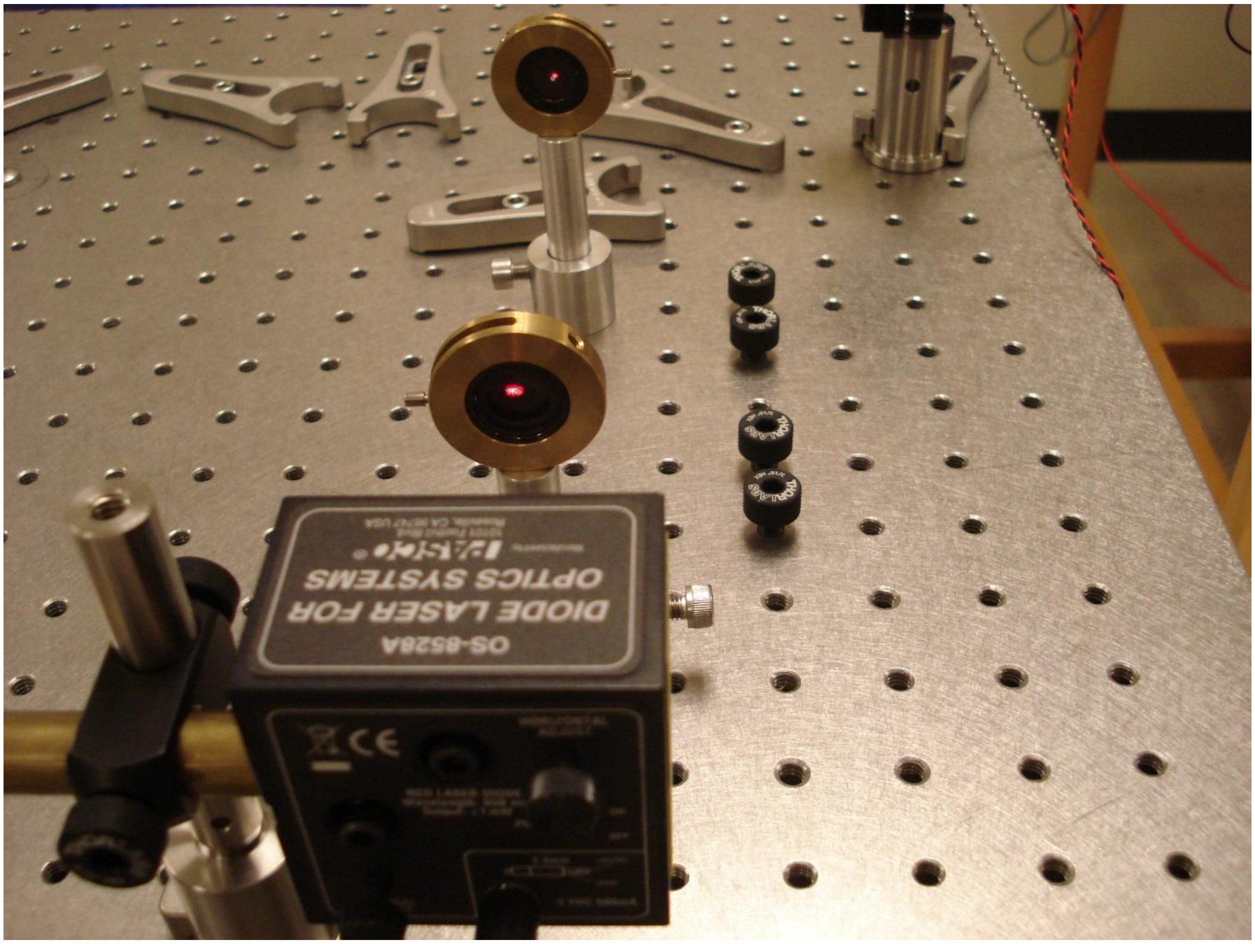

Figure 8. The HeNe beam must be at the same height as the 405 nm laser (and the two irises used with the 405 nm laser) and parallel with the tabletop.

Now position the HeNe laser and direct its beam roughly perpendicular to the 405 nm beam and approximately 3 inches in front of the 405 nm laser (Figure 9).

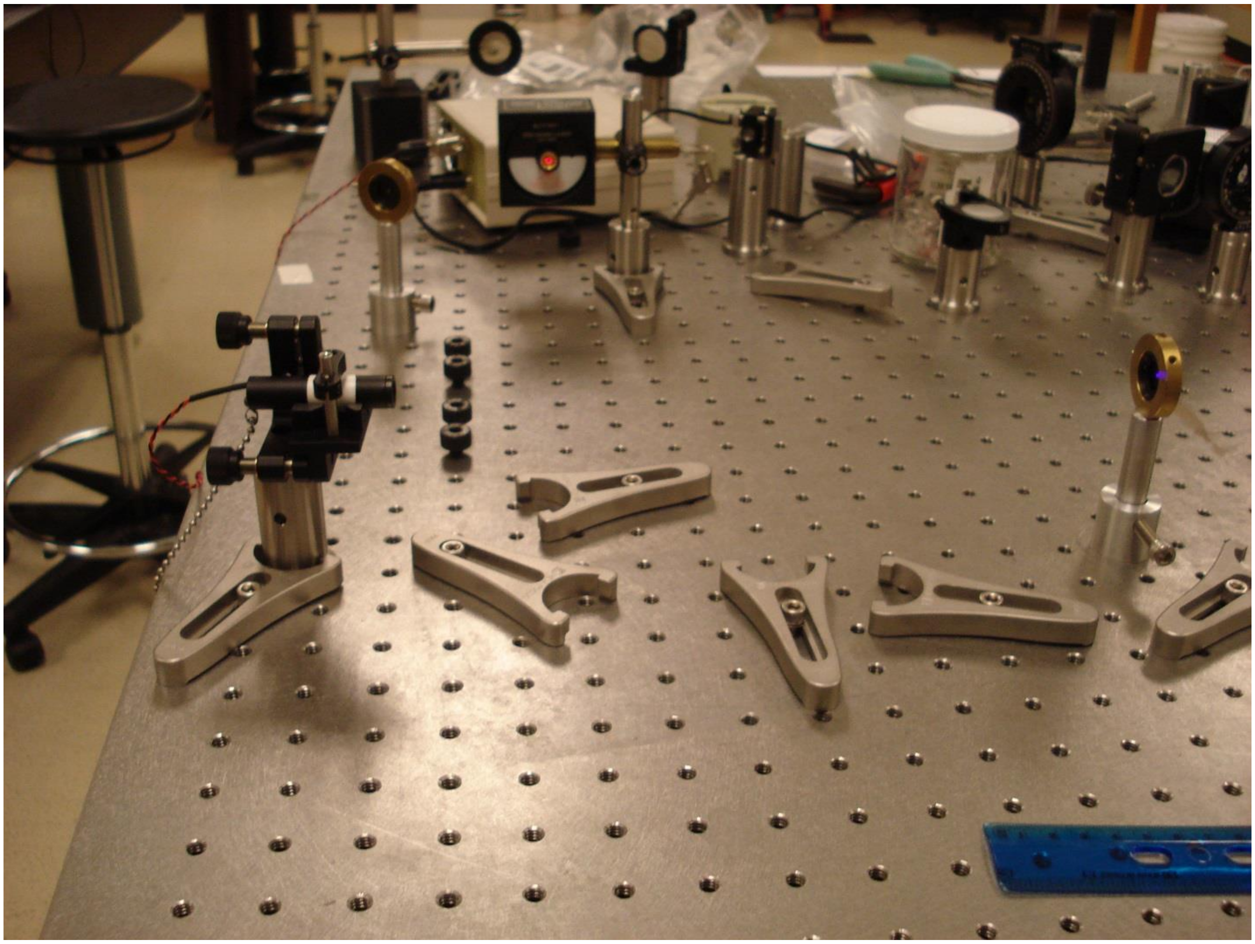

Figure 9. Setting up the HeNe beam perpendicular to the 405 nm beam.

Again have one iris where the crystal pair will go. Screw the post holder for the other iris into the bolt hole **38** inches towards the steel plate and **2** inches further from you (standing at the long edge), as shown in Figure 10. Arctan(2/38) = 3.01°, which is close enough to the 3°, the angle between the 405 nm beam and the infrared photons that will come out of the crystal pair.

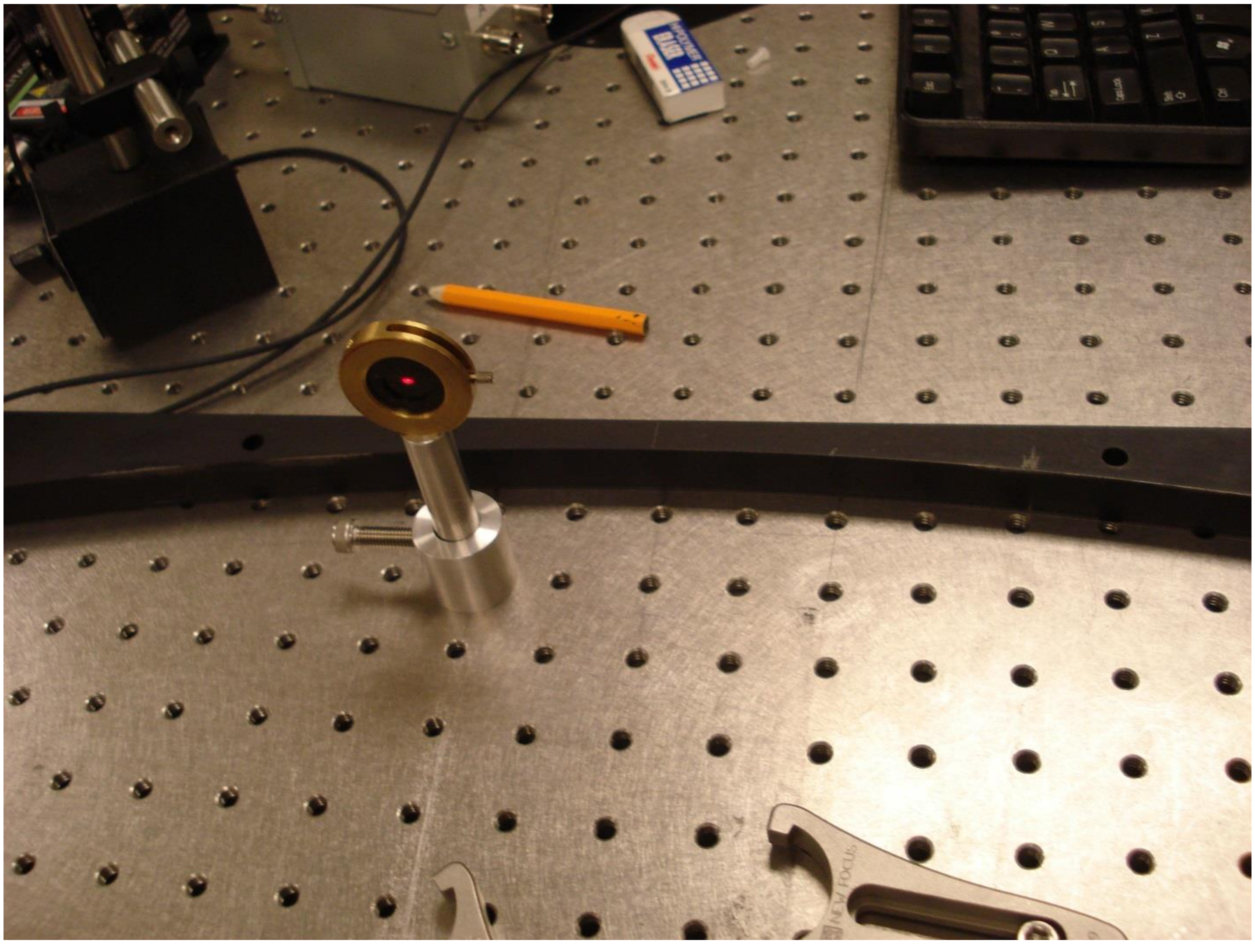

Figure 10. Positioning an iris along a path 3° from the 405 nm beam.

Position and tilt the mounted mirror so that the reflected HeNe beam goes through both irises (Figure 11). There is a risk that this mirror will get in the way of the next component. To avoid this risk, position the mirror so that the beam strikes it near its left edge, as viewed in Figure 11. Clamp the mirror in place.

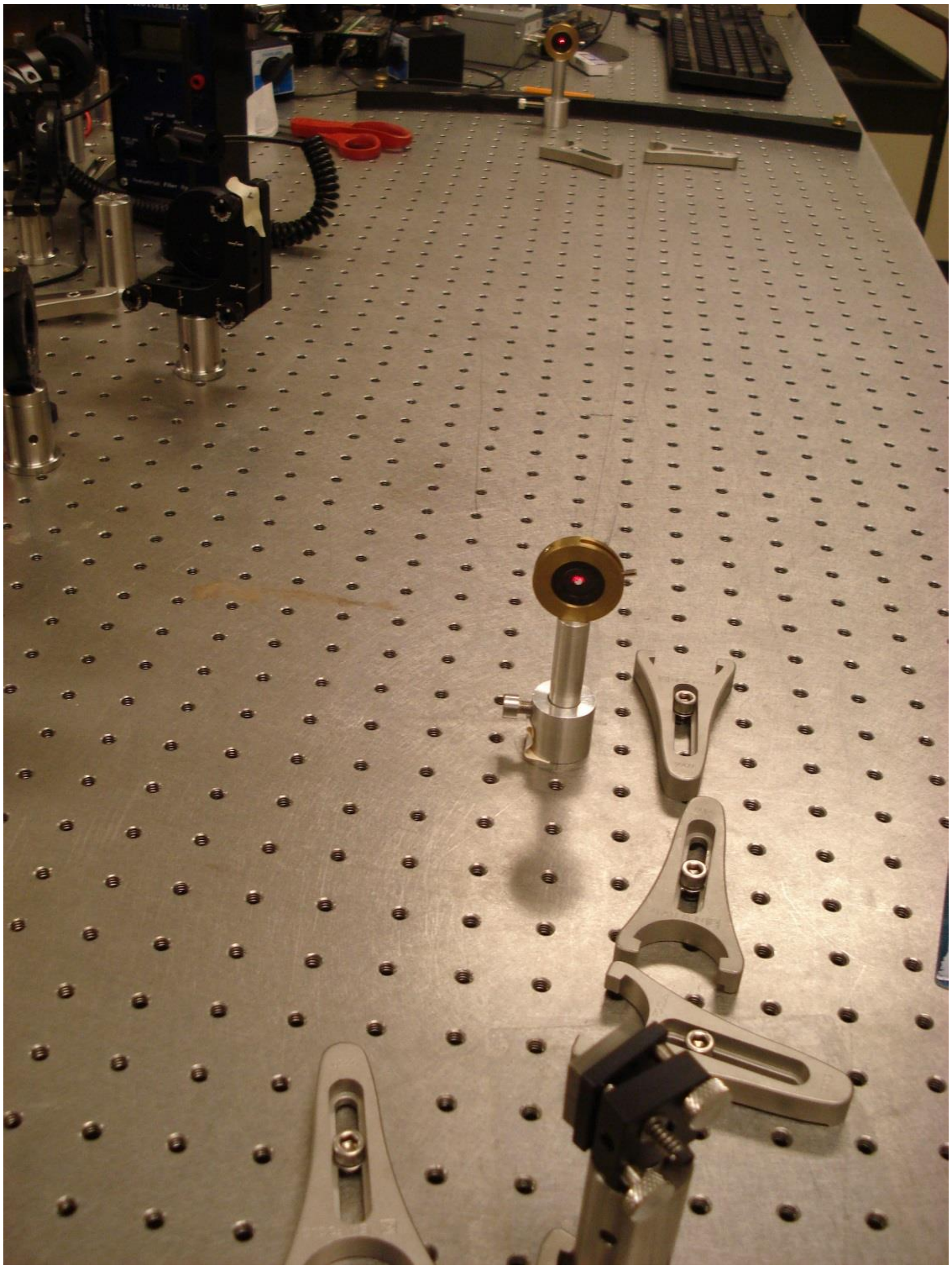

Figure 11. A mirror reflects the HeNe beam through both irises, 3° to the left of the 405 nm beam.

Now move the iris, which is by the steel plate, 4 inches so it's again 3° relative to the 405 nm beam, but in the opposite direction (Figure 12). Position and tilt the flipper mirror so the reflected HeNe laser goes through both irises. Clamp the flipper mirror in place. Remove the iris near the steel plate.

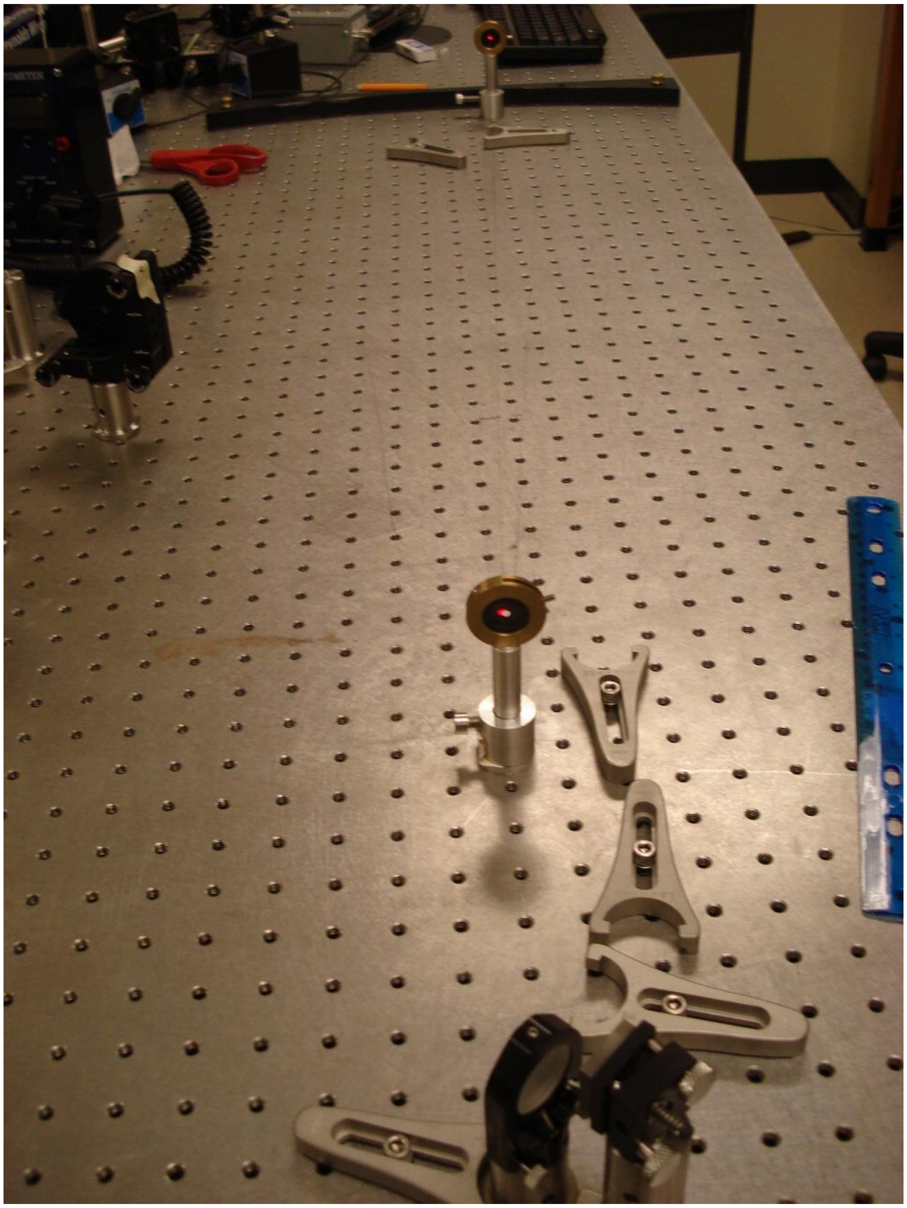

Figure 12. The flipper mirror must reflect the HeNe beam 3° to the right of the 405 nm beam.

3. Alignment of the collimators

The collimators are lenses that will focus the infrared beams onto fiber optic cables that lead to single photon detectors. Slide a collimator along the steel plate until the HeNe beam strikes the center of the iris on the collimator (Figure 13).

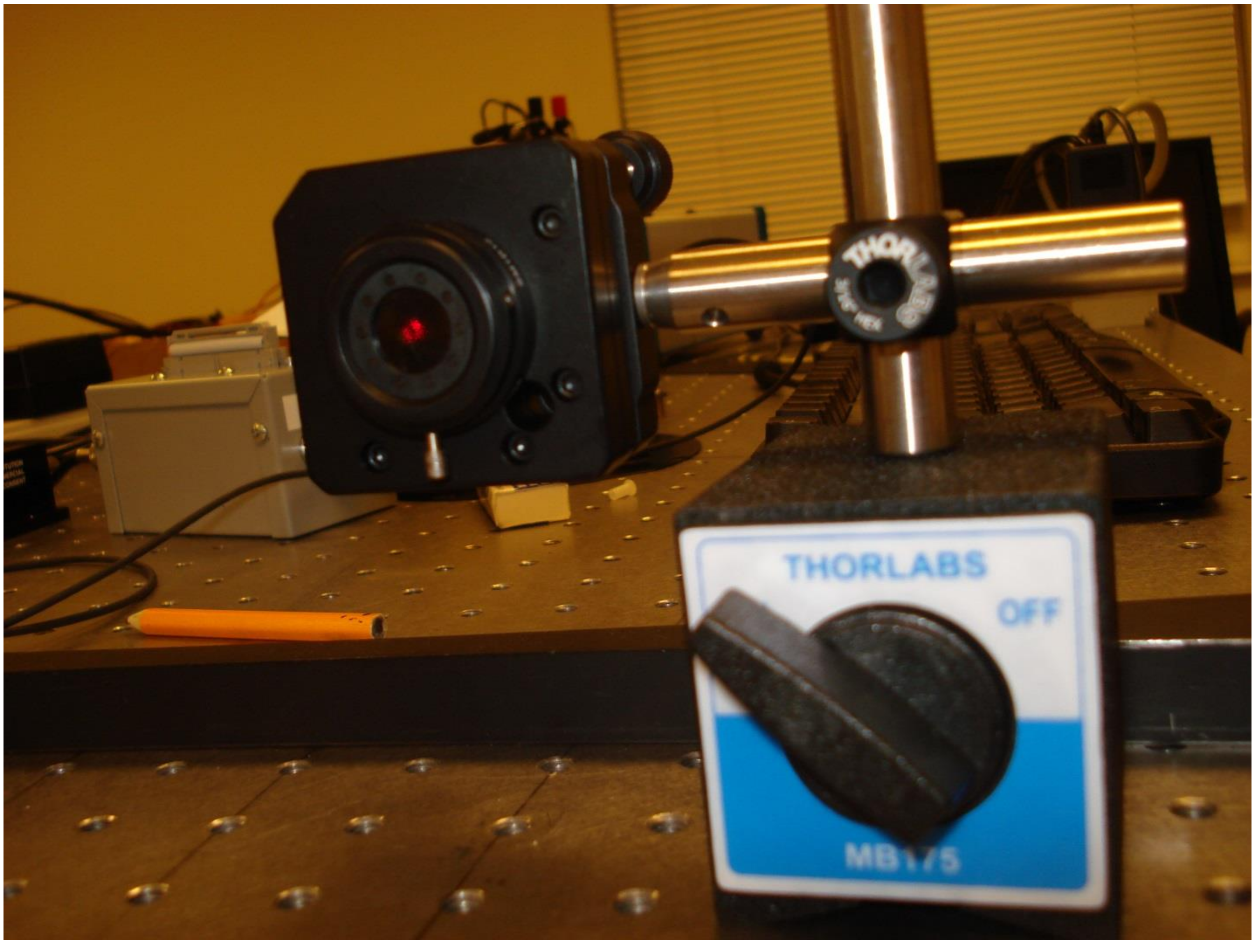

Figure 13. Aligning the collimator with the HeNe beam.

Now open that iris; we want all the light to enter the collimator.

Remove the fiber optic cable from the back of the collimator. Make sure the expanded red beam is continuing in the same direction as the HeNe beam into the collimator (Figure 14). If the expanded beam is misaligned laterally, slide the collimator along the track until the expanded beam is aligned correctly. The vertical position of the collimator may require adjustment as well.

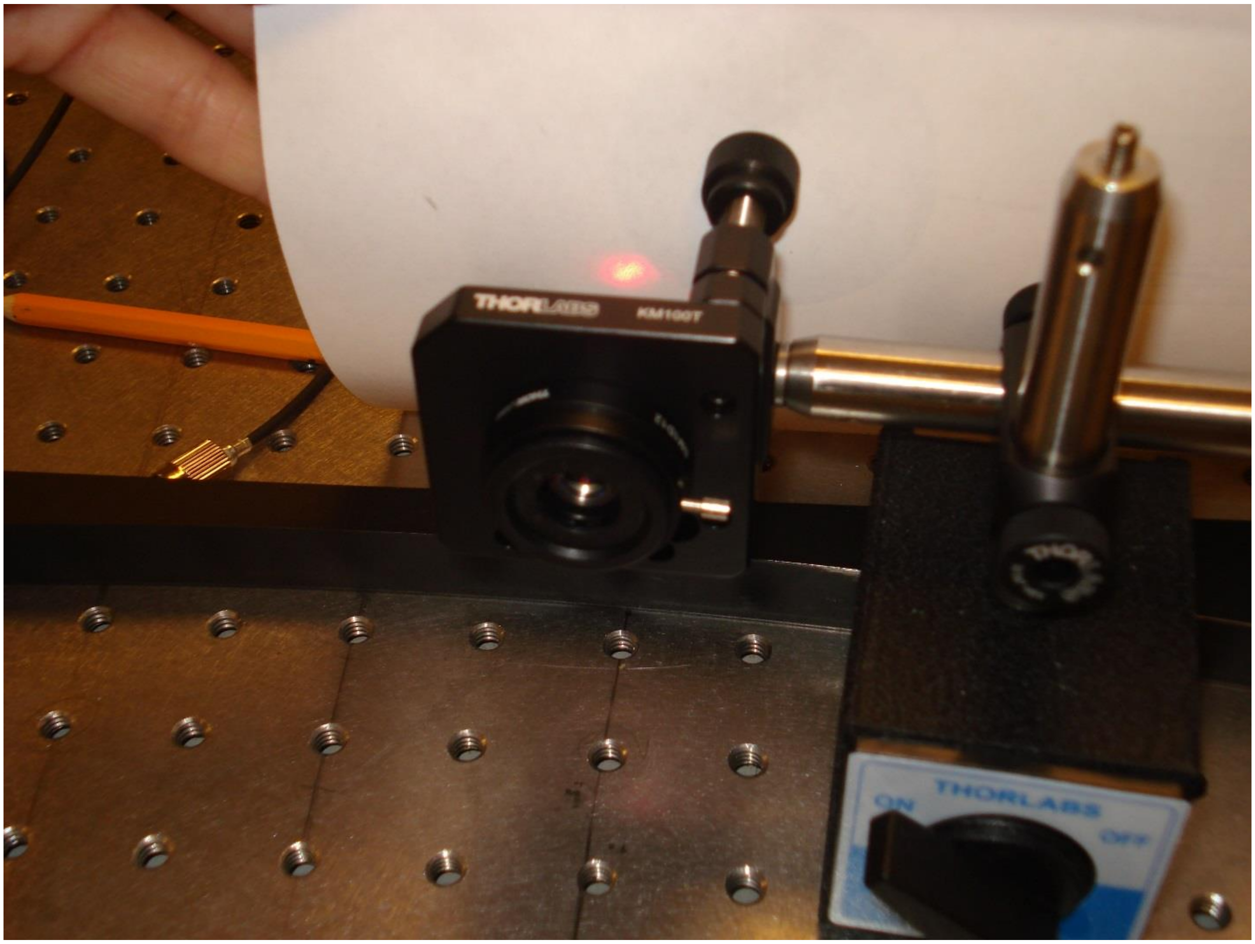

Figure 14. Observing the expanded beam behind the collimator.

Then hold the loose mirror against the front of the collimator (Figure 15).

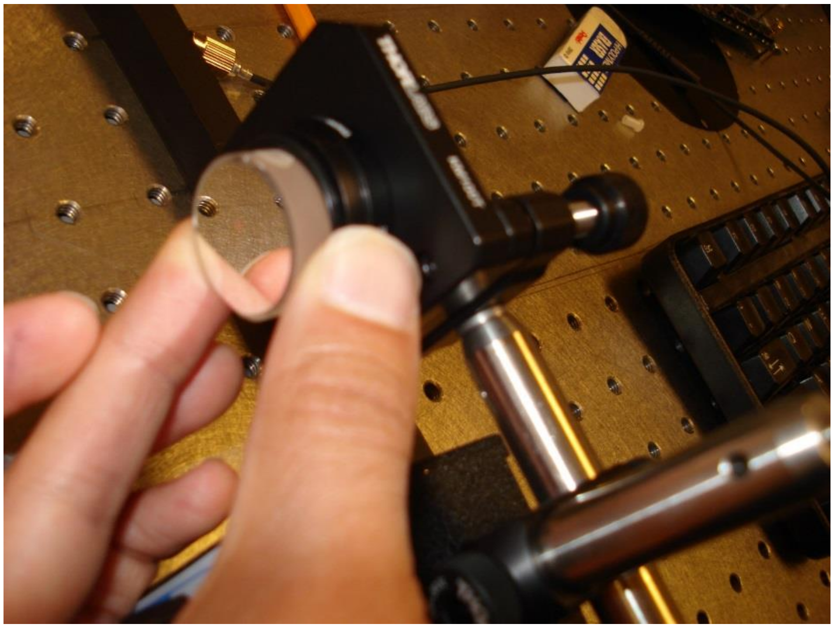

Figure 15. Placing the loose mirror against the collimator.

Adjust the tilt of the collimator until the reflected beam goes back through the iris where the crystal pair will go (Figure 16).

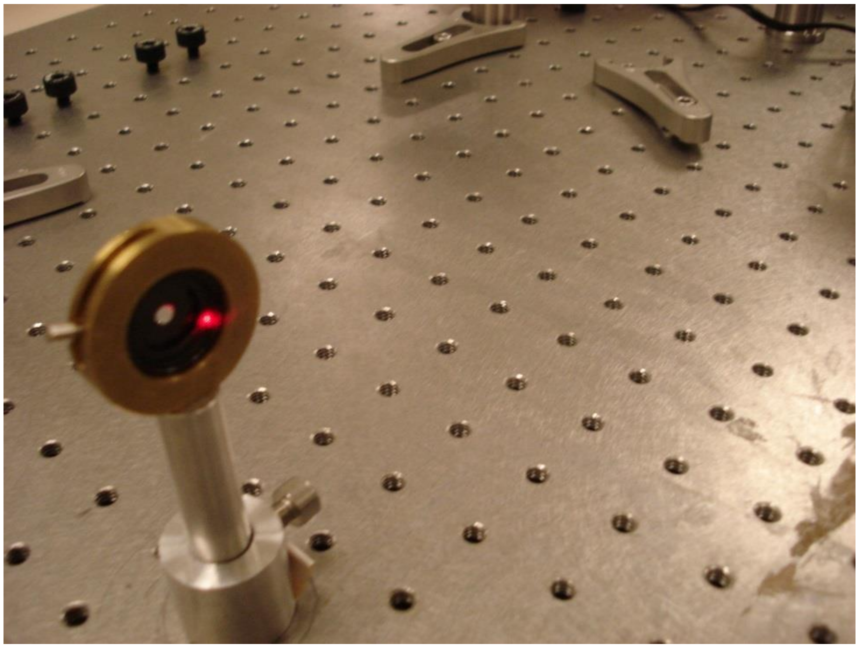

Figure 16. The tilt of the collimator must be adjusted until the beam reflecting off the loose mirror goes back through the iris.

Recheck the alignment of the expanded beam; adjust if necessary, and iterate these steps.

Reconnect the fiber optic cable to the collimator. Make sure it “clicks into place” before you tighten it; even before you tighten it, it shouldn’t be able to rotate. Disconnect the other end of the cable from the single photon detector. Observe the beam coming out of the fiber optic cable. When the cable is a few inches from the table, the beam should be bright and sharp (Figure 17). Readjust the tilt of the collimator to make the beam as bright and sharp as possible. Then reconnect the cable to the photon detector.

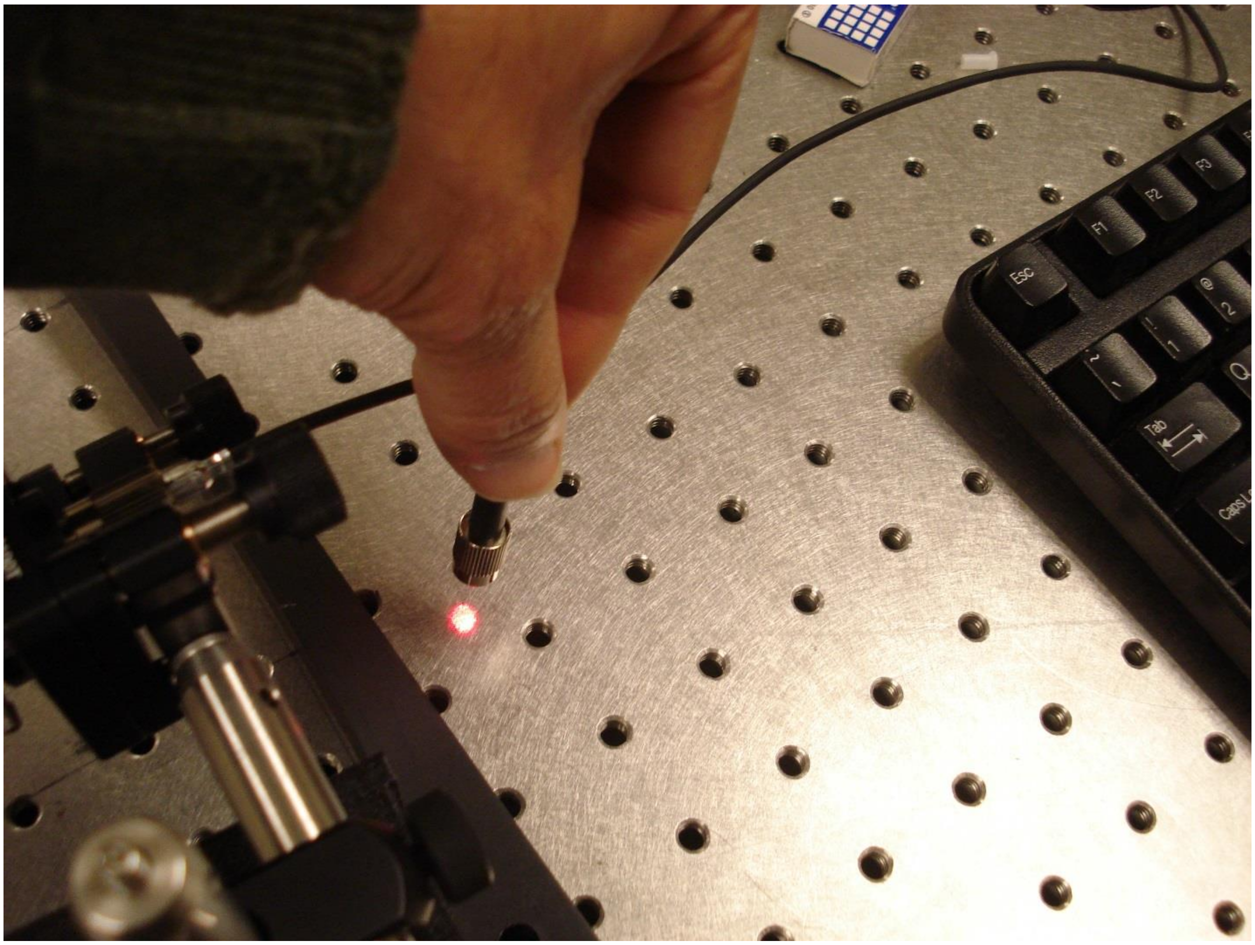

Figure 17. Observing the HeNe light coming out of the fiber optic cable.

Flip down the flipper mirror and repeat all the previous steps with the other collimator (Figure 18). When you're finished, unplug the HeNe laser. If all the alignment was done correctly, we don't need it anymore.

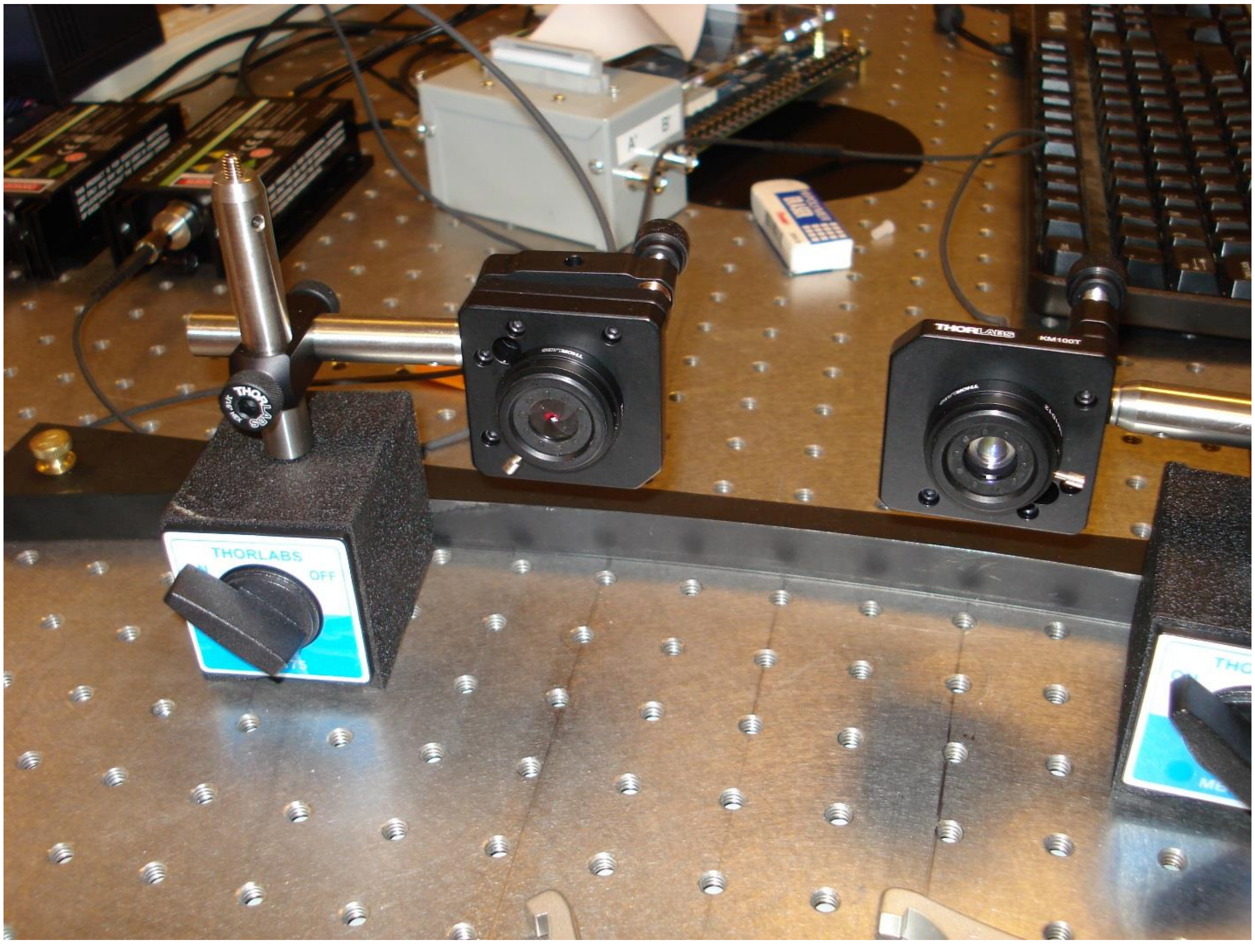

Figure 18. Repeat all the previous steps with the other collimator (flipper mirror flipped down).

4. Aligning the crystal pair

Remove the iris (and wide-headed bolt) and position the crystal pair exactly over the bolt hole (Figure 19).

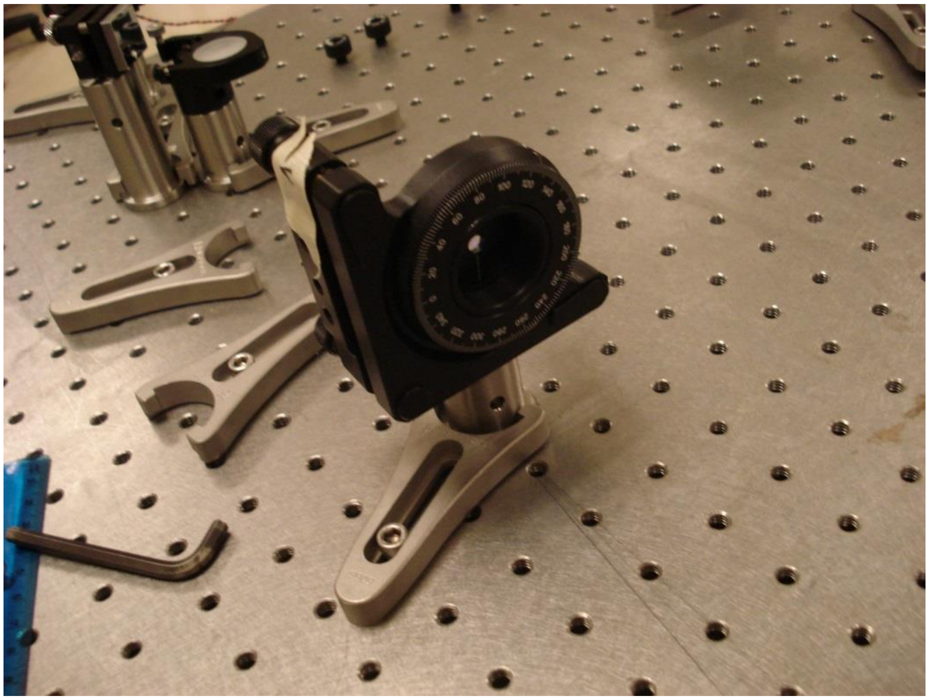

Figure 19. Clamp down the crystal pair so that the crystal pair (not the mount) is centered over the bolt hole.

We will adjust the tilt of the crystal pair separately for horizontally and vertically polarized light. The 405 nm laser is polarized, but it's difficult to rotate the laser while preserving its alignment. Therefore, we will send the 405 nm light through a half wave plate. (A half wave plate rotates the plane of polarization of linearly polarized light: the polarization is flipped about the "optical axis" of the half wave plate. As shown in Figure 20, if the optical axis makes an angle θ with the incoming polarization, the polarization is flipped 2θ.)

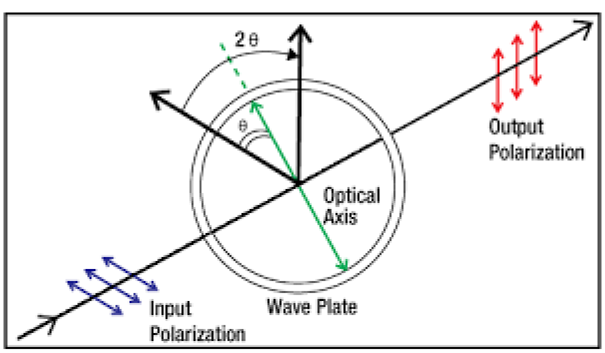

Figure 20. From Thorlabs. If the optical axis of the half wave plate makes an angle θ with the input polarization, the polarization is "flipped" by 2θ.

Place the 405 nm half wave plate between the laser and the crystal pair (Figure 21).

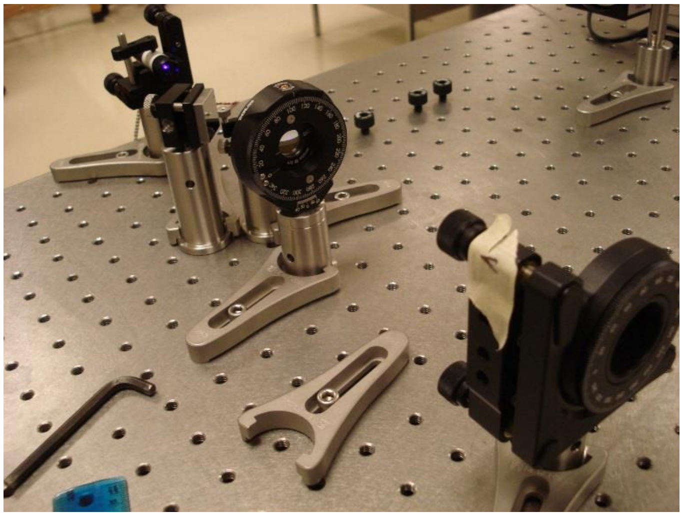

Figure 21. The half wave plate is placed between the laser and the crystal pair.

You can rotate the half wave plate to change the polarization of the 405 nm light. (You should send the light through a known polarizer and into a photometer to observe how final intensity depends on the angle of the wave plate.) Rotate the half wave plate so that the output polarization is horizontal. (When I did this, I needed to set the half wave plate to 44°, but if someone rotated the laser in the holder, it's different now.)

Now place filters in front of the collimators to block as much visible light as possible (Figure 22).

Figure 22. These optical bandpass filters block all light except wavelengths near 810 nm. These do not have to be clamped in place.

Turn on the field programmable gate array (FPGA, Figure 23). The FPGA contains the electronic circuit that counts the voltages pulses produced by the single photons detectors. (The single photon detector outputs a voltage pulse every time a single photon arrives at the detector.)

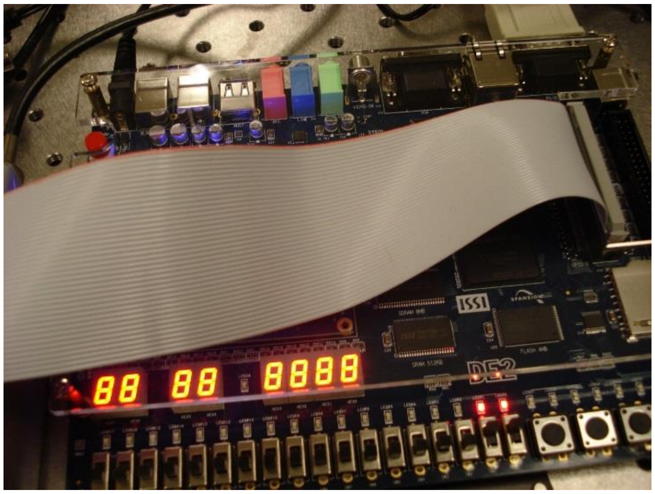

Figure 23. Push the red button to turn this device on. It contains that circuit that counts the voltage pulses output by the single photon detectors.

Open the MATLAB script that will display the data.

TURN OFF THE ROOM LIGHTS BEFORE YOU TURN ON THE SINGLE PHOTON DETECTORS! THE SINGLE PHOTON DETECTORS WILL BE TEMPORARILY DISABLED AND POSSIBLY DESTROYED IF TURNED ON WHILE THE ROOM LIGHTS ARE ON! Once you're in the dark, turn on the power for the single photon detectors (Figure 24), but BE SURE TO TURN IT OFF BEFORE YOU TURN ON THE ROOM LIGHTS!

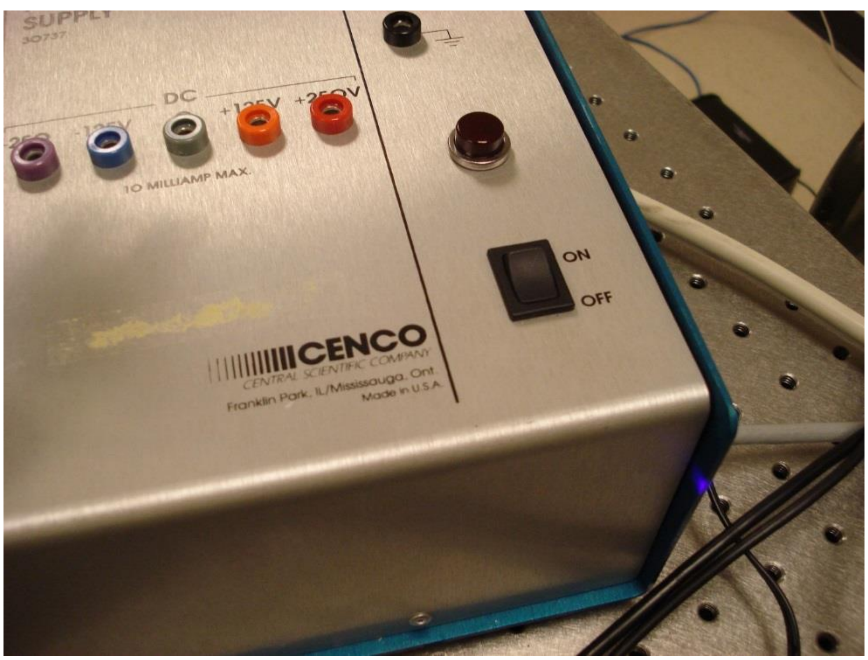

Figure 24. Turn this on **only in the dark! Always turn this off** before turning the room lights back on!

In the darkness, with the FPGA and single photon detectors turned on, run the MATLAB script. You should see three graphs: two showing counts/second for each of the two detectors (these are called “singles” counts), and also coincidences/second. A **coincidence** occurs when photons are detected **simultaneously** at the two detectors. **All we really care about is coincidences** because when a 405 nm photon splits in a BBO crystal, it produces two 810 photons which, if they’re in a horizontal plane, will arrive **simultaneously** at the two detectors.

If the alignment is good, you should be getting about 100 coincidences/second at this point. Adjust the **horizontal** tilt of the crystal pair to maximize coincidences. (We’re sending in horizontally polarized 405 nm light, which produces vertically polarized 810 nm light. It’s the horizontal tilt of the crystal pair that tilts the cone of vertically polarized 810 nm light.) As you adjust the tilt, the singles counts are affected as well as the coincidences. You should see both singles counts peak together.

If you’re getting about 100 coincidences/second (or more), you can leave well enough alone. If the coincidences are much lower, there are two other things you can try to increase coincidences: first, adjust the tilt of the collimators. Next, if you still want to try to increase coincidences, you can try slightly (by less than 1 mm) sliding one of the collimators along the steel plate.

We’ve adjusted the horizontal tilt of the crystal pair. Next, rotate the half wave plate to produce vertically polarized light (the half wave plate was at 359° when I did it). We want the coincidences to be the same as they just were for the other polarization. If they’re not, adjust the **vertical** tilt of the crystal pair until the coincidences are about the same for both polarizations.

5. Aligning the compensating crystal

Now we need the 405 nm photons to be polarized at 45° so that there are equal components of horizontal and vertical polarization. Rotate the 405 nm half wave plate to produce 45° polarization (I needed the half wave plate to be at 21.5°). Now place high quality polarizers (Glan-Thompson polarizers) in front of the collimators (Figure 25). They should be set for vertical polarization. By observing coincidences (the room is dark, of course!), make sure you’re not blocking the beam with the polarizers’ housing.

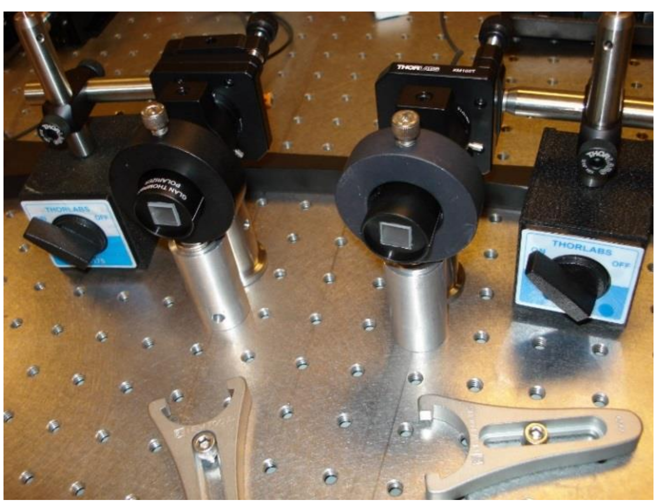

Figure 25. Position these polarizers carefully in the dark to avoid blocking any of the infrared beam with the polarizers’ housing.

Next, place half wave plates in front of the polarizers (Figure 26).

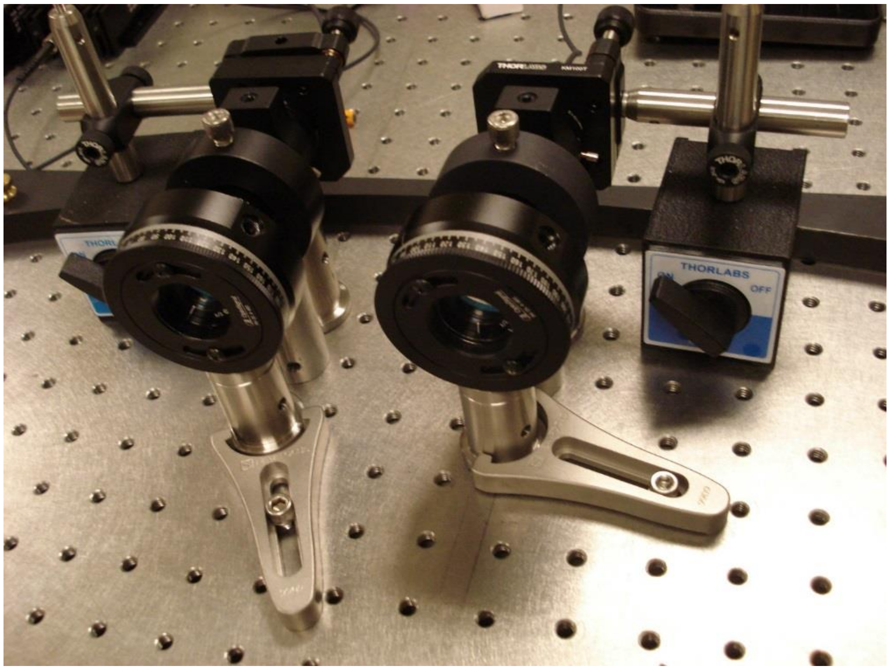

Figure 26. Half wave plates are used in combination with the fixed polarizers to create, in effect, rotating polarizers. (If you turn the lights on to position these, remember to shut off the single photon counters first!)

The polarizers have a square cross section. If the infrared beam (which we can't see) is travelling through the square corner of a polarizer, we'll block the beam if we rotate the polarizer. So we'd better not rotate the polarizers. We use half wave plates to rotate the beam polarization instead. For example, if we want to measure horizontally polarized photons, we set the half wave plates to 45°. This flips horizontally polarized photons 90°, to vertical, and these photons will pass through the vertical polarizers.

You should see that when both half wave plates are 0°, you get about the same coincidences as when they're both 45°. (If not, the crystal pair was not tilted properly.) Coincidences should be minimized when one half wave plate is 0° and the other is 45°. (Why? It's important to understand this.)

Glance back (all the way back!) at Eq. (1). We need to make sure the horizontally polarized "component" is in phase with the vertically polarized component. However, horizontally polarized infrared photons are produced in one of the two crystals, and vertically polarized infrared photons are produced in the other one. One of the crystals comes before the other. The infrared photons produced in the first crystal must pass through the second crystal; those produced in the second crystal do not pass through any another crystal. This difference creates a phase shift that we preemptively reverse by use of a compensating crystal.

Place the compensating crystal between the 405 nm half wave plate and the crystal pair (Figure 27).

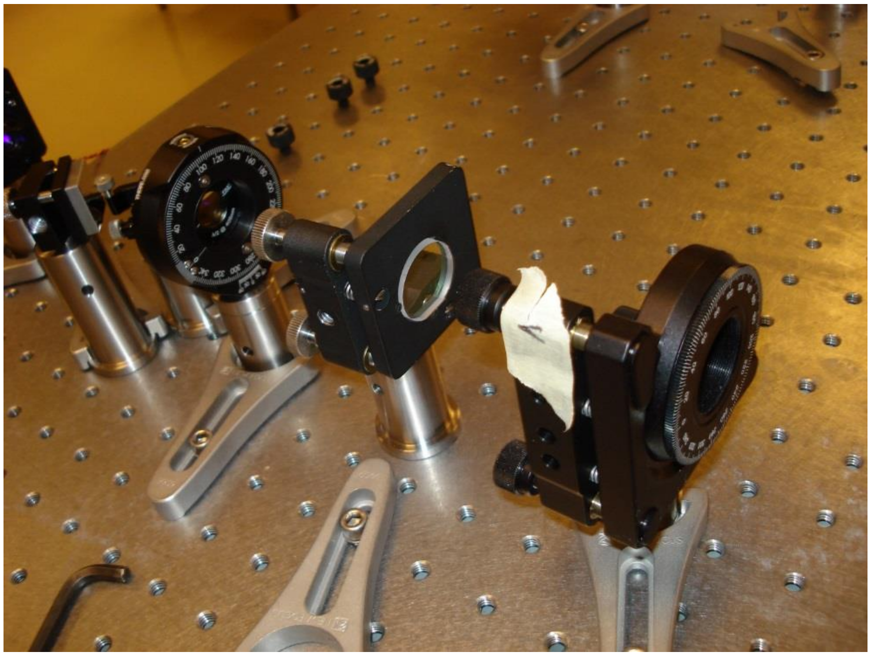

Figure 27. The compensating crystal goes between the half wave plate and the crystal pair. Remember, the room light must be off if the single photon detectors are on!

You've already achieved the following:

- Maximum and ~equal coincidences when the 810 nm half wave plates are both 0° or both 45°.
- Minimum (~0) coincidences when one half wave plate is 0° and the other is 45°.

Adjust the tilt of the compensating crystal to achieve the following:

- Maximum and ~equal coincidences when the half wave plates are both 22.5° or both 67.5° (same maximum as for 0° or 45°).
- Minimum (~0) coincidences when one half wave plate is 22.5° and the other is 67.5°.

I found that if I tilted the compensating crystal to get the same maximum at 22.5° as I had for 0° and 45°, the other criteria were automatically satisfied. If all four bullet points are satisfied, you've achieved Eq. (1) and are ready to investigate quantum entanglement!

6. Test of a Bell inequality

We'll discuss the theory below. It's a little abstract, so I'll first describe the procedure, which is rather concrete and simple, now that you've completed the alignment.

You'll record coincidences for 16 combinations of angles of the 810 nm half wave plates. You can arbitrarily decide which plate is A and which is B; it doesn't matter.

	Half wave plate A (°)	Half wave plate B (°)
1.	22.5	11.25
2.	22.5	56.25
3.	67.5	11.25
4.	67.5	56.25
5.	22.5	33.75
6.	22.5	78.75
7.	67.5	33.75
8.	67.5	78.75
9.	0	11.25
10.	0	56.25
11.	45	11.25
12.	45	56.25
13.	0	33.75
14.	0	78.75
15.	45	33.75
16.	45	78.75

Set the half wave plates to appropriate angles, and then click the corresponding blank in the table in the MATLAB GUI (graphical user interface). The number of coincidences should automatically appear in the table. After you've completed the table, click Calculate. The script will calculate a quantity called S, and its uncertainty σ_S, so your result is $S \pm \sigma_S$. If $S > 2$, you've violated a Bell inequality, which is the whole point of this experiment. (Theoretical details below.) If $S > 2$, but $S - \sigma_S < 2$, your results are inconclusive, and you need to reduce the uncertainty. You can do this by increasing the duration of each measurement. You can try 5 seconds (50 tenths of a second, so you enter 50 in the GUI). If you use 5 seconds, be sure the coincidences are recorded in a time interval **after** you stop adjusting the half wave plates.

Record your results for S and σ_S. Also copy the 16 coincidence values in your table.

There's a glitch in the script: after you click Calculate, you have to exit and restart the script before taking additional measurements.

7. Malus's law for entangled photons

Keep one of the half wave plates fixed, and record coincidences while the other one is varied in 5° increments over a 90° range. To reduce fractional error, the duration of each measurement should be at least 5 s.

THEORY

1. Definitions

First we have to understand that when a half wave plate is set to angle A relative to the vertical, the output polarization will be vertical if the input polarization is 2A. (Refer back to Figure 20, and modify it appropriately.) So **setting a half wave plate to angle A (in front of a vertical polarizer) is equivalent to rotating the polarizer to an angle 2A**. Let's define α = 2A, and similarly β = 2B. Let's call α and β **measurement angles**.

Next, let's define $N(\alpha,\beta)$ as the coincidences recorded when one half wave plate is set to A = $\alpha/2$, and the other is set to B = $\beta/2$. Let's next define a quantity

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha+90^\circ,\beta+90^\circ) - N(\alpha,\beta+90^\circ) - N(\alpha+90^\circ,\beta)}{N(\alpha,\beta) + N(\alpha+90^\circ,\beta+90^\circ) + N(\alpha,\beta+90^\circ) + N(\alpha+90^\circ,\beta)}. \quad (4)$$

Finally, we define

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b'), \quad (5)$$

where we choose a = 0°, b = 22.5°, a' = 45°, and b' = 67.5°.

We need to rewrite Eq. (4) in terms of probabilities. If we know N_{tot}, the total number of photon pairs arriving at the detectors in a chosen time interval, then we can write the probability that both photons are Transmitted (so that we detect them),

$$P_{TT}(\alpha,\beta) = P(\alpha,\beta) = N(\alpha,\beta)/N_{tot}. \quad (6)$$

The probability that both photons are Extinguished (absorbed by the polarizers) is the probability that, if the measurement angles changed by 90°, the photons would be transmitted (and detected):

$$P_{EE}(\alpha,\beta) = P(\alpha+90^\circ,\beta+90^\circ) = N(\alpha+90^\circ,\beta+90^\circ)/N_{tot}. \quad (7)$$

To make sense of Eq. (7), it's helpful to refer to Eq. (2). We can always decompose polarization into two perpendicular components. We can never directly measure the component that gets extinguished. Eq. (7) just says that the **extinguished components become transmitted components if we rotate the measurement angles 90°.**

By similar logic, the probabilities that one photon is transmitted and one is extinguished are

$$P_{TE}(\alpha,\beta) = P(\alpha,\beta+90^\circ) = N(\alpha,\beta+90^\circ)/N_{tot} \quad (8)$$

and

$$P_{ET}(\alpha,\beta) = P(\alpha+90^\circ,\beta) = N(\alpha+90^\circ,\beta)/N_{tot}. \quad (9)$$

Since each photon is either transmitted or extinguished, there are four possibilities for the two photons in the pair, and the sum of the probabilities is 1:

$$P_{TT}(\alpha,\beta) + P_{EE}(\alpha,\beta) + P_{TE}(\alpha,\beta) + P_{ET}(\alpha,\beta) = 1. \quad (10)$$

Plugging Eqs. (6)-(10) into Eq. (4) yields

$$E(\alpha,\beta) = P_{TT}(\alpha,\beta) + P_{EE}(\alpha,\beta) - P_{TE}(\alpha,\beta) - P_{ET}(\alpha,\beta). \quad (11)$$

2. Bell inequality

Suppose each photon has properties all along that predetermine whether it will pass through a polarizer at any chosen angle. The polarizer, in effect, "reads off" the photon's property, which we call a "hidden variable." The hidden variable is represented by λ, and its probability density is $\rho(\lambda)$. For example, if $\lambda = 0$ half the time and $\lambda = \pi/2$ the other half of the time, then $\rho(0) = .5$ and $\rho(\pi/2) = .5$, and $\rho(\lambda) = 0$ for all other λ.

Now, let's define a function $A(\lambda,\alpha)$ for the photon traveling to the polarizer set to angle α:

$A(\lambda,\alpha) = +1$ if the photon is transmitted (detected at measurement angle α).

$A(\lambda,\alpha) = -1$ if the photon is extinguished (not detected at measurement angle α).

The function $B(\lambda,\beta)$ is similarly defined. These functions effectively determine what a photon will do, based on its own hidden variable and the measurement angle of the polarizer it approaches. If you want a concrete and simple example, suppose $A(\lambda,\alpha) = +1$ if $|\alpha-\lambda| < 45°$ or $> 135°$; otherwise $A(\lambda,\alpha) = -1$. This says the photon transmitted if its "hidden angle" λ is closer to α (or $\alpha\pm180°$) than it is to $\alpha\pm90°$. (I'm assuming $|\alpha-\lambda|$ is between 0° and 180°.)

In fact, we don't know $A(\lambda,\alpha)$, $B(\lambda,\beta)$, or $\rho(\lambda)$. But **if** each photon has properties that predetermine the result of a measurement, then these functions must exist, even though we don't know what they are.

Notice that

$$\frac{1+A(\lambda,\alpha)}{2}$$

is 1 if the photon heading toward A is transmitted and 0 if it's not. Similarly,

$$\frac{1+B(\lambda,\beta)}{2}$$

is 1 if the other photon is transmitted and 0 if it's not. Let's multiply these expressions:

$$\frac{1+A(\lambda,\alpha)}{2}\frac{1+B(\lambda,\beta)}{2}.$$

This is 1 if both photons are transmitted and 0 otherwise. At some values of λ, this expression may be 1 (the photons are both transmitted); at other values of λ, this expression may be 0 (the photons are not both transmitted). So if we average this expression over all values of λ, we get the probability that both photons are transmitted:

$$P_{TT}(\alpha,\beta) = \int \frac{1+A(\lambda,\alpha)}{2}\frac{1+B(\lambda,\beta)}{2}\rho(\lambda)d\lambda\,. \qquad (12)$$

By similar logic,

$$P_{EE}(\alpha,\beta)=\int\frac{1-A(\lambda,\alpha)}{2}\frac{1-B(\lambda,\beta)}{2}\rho(\lambda)d\lambda\,, \tag{13}$$

$$P_{TE}(\alpha,\beta)=\int\frac{1+A(\lambda,\alpha)}{2}\frac{1-B(\lambda,\beta)}{2}\rho(\lambda)d\lambda\,, \tag{14}$$

and

$$P_{ET}(\alpha,\beta)=\int\frac{1-A(\lambda,\alpha)}{2}\frac{1+B(\lambda,\beta)}{2}\rho(\lambda)d\lambda\,. \tag{15}$$

In your lab report, you'll combine Eqs. (11)-(15) to derive

$$E(\alpha,\beta)=\int A(\lambda,\alpha)B(\lambda,\alpha)\rho(\lambda)d\lambda\,. \tag{16}$$

Next, we define

$$s = A(\lambda,a)[B(\lambda,b) - B(\lambda,b')] + A(\lambda,a')[B(\lambda,b) + B(\lambda,b')] \tag{17}$$

where a, b, a', and b' are four angles, as in Eq. (5). Notice that since the functions A and B are always 1 or -1, there are only two possible values of s: 2 and -2.

Averaging Eq. (17) over all λ, the average value of s (which is what we expect to obtain experimentally over sufficiently long time intervals) is

$$S=\int s\rho(\lambda)d\lambda\,. \tag{18}$$

In your lab report, you will show that this leads to Eq. (5).

Since s can be only 2 or -2, S (the average of s) must satisfy $-2 \le S \le 2$. This is the Clauser-Horne-Shimony-Holt version of a **Bell inequality**. If your data violates this inequality, then one of the assumptions leading to it must be wrong. The wrong assumption is that each photon has properties all along that determine the outcome of a measurement. Instead, prior to measurement, the photons are in the state given by Eq. (1). Not only is each photon in a superposition of horizontal and vertical polarization, but the two photons are entangled: when the polarization of one photon is measured, the other photon (for all practical purposes) immediately acquires the same polarization. For example, if one photon passes through a horizontal polarizer, we immediately know that the other photon will also pass through a horizontal polarizer; the other photon wasn't horizontal all along, but was in a superposition of horizontal and vertical polarization. Does the measurement of one photon physically alter the other photon, or is something subtler happening? There are many answers to this question and many interpretations of quantum mechanics.

In your lab report, you will show that **quantum mechanics predicts S > 2** for the chosen angles. If you measure S > 2, then you have experimental evidence that supports quantum mechanics, and that contradicts a theory of hidden variables (which assumes the photons all along have properties that determine measurement outcomes).

To compute σ_S, the uncertainty in S, we need to recognize that S is based on 16 measurements of coincidence counts. I'll call the coincidence data N_i, where i varies from 1 to 16. According to Poisson statistics, the uncertainty in N_i is $\sqrt{N_i}$. The rules of error propagation therefore give

$$\sigma_S = \sqrt{\sum\left(\sigma_{N_i}\frac{\partial S}{\partial N_i}\right)^2} = \sqrt{\sum N_i\left(\frac{\partial S}{\partial N_i}\right)^2} \,. \quad (19)$$

This calculation is performed automatically in the MATLAB script.

IN YOUR LAB REPORT

Theory (to include in lab report):

1. Quantum mechanical probabilities

Prove that Eq. (3) is ½cos²(*β*-*α*). Hints:

Preliminary information: First let's think about just one photon. Suppose a photon is in a state

$$\frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right),$$

which is equivalent to 45° polarization. This photon has a 50% chance of passing through a horizontal polarizer. In general, what's the probability that a photon is transmitted through a horizontal polarizer? Answer: The probability is the square of the $|H\rangle$ coefficient, in our case, $\left(\frac{1}{\sqrt{2}}\right)^2$. Specifically, we perform the following steps:

- We multiply the state by $\langle H|$ and use the rules $\langle H|H\rangle = 1$ and $\langle H|V\rangle = 0$. These are exactly analogous to the unit vector rules $\hat{i}\bullet\hat{i} = 1$ and $\hat{i}\bullet\hat{j} = 0$.
- We compute the square of the absolute value of the result. In our 45° example, we obtain $\left|\langle H|\frac{1}{\sqrt{2}}\left(|V\rangle + |H\rangle\right)\right|^2$. Can you see why this is 50%?

Things are just a little more complicated when we have two particles to keep track of. The rules to follow are

$$\langle H|_A|H\rangle_A = 1,\ \langle V|_A|V\rangle_A = 1, \langle H|_B|H\rangle_B = 1, \langle V|_B|V\rangle_B = 1,$$

$$\langle H|_A|V\rangle_A = 0, \langle V|_A|H\rangle_A = 0, \langle H|_B|V\rangle_B = 0, \langle V|_B|H\rangle_B = 0.$$

A symbol with an A subscript acts as a coefficient multiplying a symbol with a B subscript, and vice versa. So, for example, $\left(\langle V|_A\langle H|_B\right)\left(|H\rangle_A|H\rangle_B\right) = \langle V|_A|H\rangle_A\langle H|_B|H\rangle_A = 0\times 1 = 0$.

2. Bell inequality

Fill in the steps omitted in the derivation: Derive Eq. (16), explain why s can only be 2 or -2, and show that Eqs. (16)-(18) lead to Eq. (5).

3. Quantum mechanical prediction for S

Use Eqs. (3) and (5)-(11) to determine the quantum mechanical prediction for S. Experimental results are lower than this prediction because of the effect of accidental coincidences (which occur due to the finite time resolution of the single photon detectors).

<u>Experiment (to include in lab report):</u>

Show your table of results for the 16 combinations of half wave plate angles. What time interval did you use? What is your final result, $S \pm \sigma_S$? Did you obtain a Bell inequality violation? What is the significance of this result?

In the Malus's law experiment, you varied one half wave plate while keeping the other fixed. Plot coincidences as a function of β (or α, whichever was varying). Compare with Eq. (3).

REFERENCES

Dehlinger and Mitchell, Am. J. Phys. **70**, 903-910 (2002). Their angles are relative to the vertical, and mine are relative to the horizontal.

http://departments.colgate.edu/physics/research/Photon/root/lab5entanglement13.pdf

Griffiths, *Introduction to Quantum Mechanics*: the Afterword and references therein.

CLEAN-UP

Beta barium borate is hygroscopic (it absorbs moisture). To keep it from fogging up, put the crystal pair in the jar of desiccant when you complete your measurements. Please be careful not to let the crystal pair (the small circle) touch anything! Dr. Seuss describes best the way I felt the first time I disassembled the apparatus:

Suppose, just suppose, you were poor Herbie Hart,
who has taken his Throm-dim-bu-lator apart!
He never will get it together, I'm sure.
He never will know if the Gick or the Goor
fits into the Skrux or the Snux or the Snoor.
Yes, Duckie, you're lucky you're not Herbie Hart
who has taken his Throm-dim-bu-lator apart!

--Dr. Seuss