Measuring Input and Output Resistance

Introduction

The objective of these notes is to allow you to quickly and accurately measure the input and output resistance of circuit elements so that you can concentrate on more advanced topics.

Preliminary Note about Circuit Diagrams

You're probably used to seeing circuit diagrams like this:

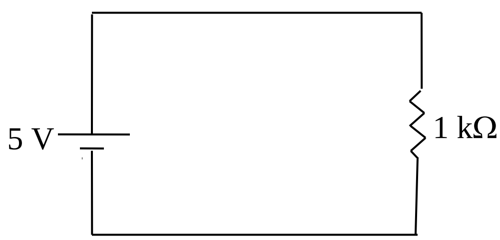

Figure 1. A circuit diagram in the style of an introductory physics textbook.

However, electronics textbooks follow a different convention:

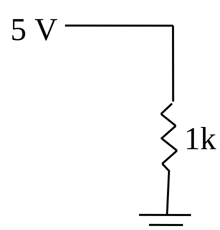

Figure 2. The same circuit in the style of an electronics textbook.

Figures 1 and 2 both represent exactly the same circuit! It's a 1k resistor with 5 V across it. Figure 2 takes a little less time to draw, so we prefer Figure 2.

Input Resistance

Suppose you construct the following circuit:

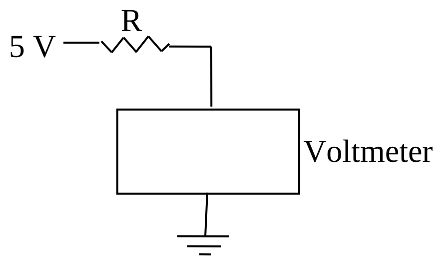

Figure 3. A voltage source connected to a voltmeter through a resistor.

QUESTION: Does the voltage recorded by the voltmeter depend on R?

WRONG answer: No current ever flows into a voltmeter, so the current through R is 0, so the voltage across R is 0, so the voltmeter always sees 5 V across its terminals.

RIGHT answer: Very little current flows into the voltmeter because its input resistance (R_{in}) is very high, but R_{in} is not infinite in a real voltmeter. R_{in} appears in the circuit as follows:

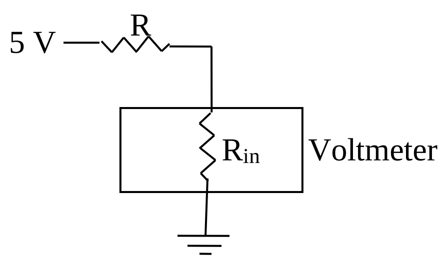

Figure 4. The input resistance of the voltmeter is revealed.

When R << R_{in}, R_{in} dominates the series combination of R and R_{in}, which means that nearly the full 5 V appears across R_{in}. However, when R grows nearly as large as R_{in}, we can no longer neglect R. The current through the series combination is (5 V)/(R + R_{in}), so the voltage across R_{in} (which is what the voltmeter measures) is

$$V_{measured} = 5V \frac{R_{in}}{R + R_{in}}. \qquad (1)$$

You can see from Equation (1) that $V_{measured}$ approaches 5 V as R gets small, but $V_{measured}$ drops as R approaches R_{in}.

It seems that the higher R_{in} is, the better; higher R_{in} values allow R to be higher before $V_{measured}$ is affected. **High input resistance is good.**

Output Resistance

Now consider the following circuit:

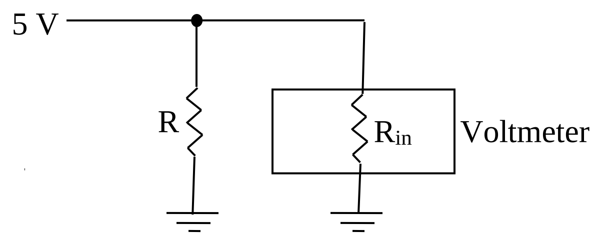

Figure 5. A voltage source connected to the parallel combination of a resistor and a voltmeter.

QUESTION: Does the voltage recorded by the voltmeter depend on R?

WRONG answer: Evidently, the voltage at one end of the voltmeter is always 5 V, and the voltage at the other end is always ground.

RIGHT answer: We must consider the output resistance of the voltage source:

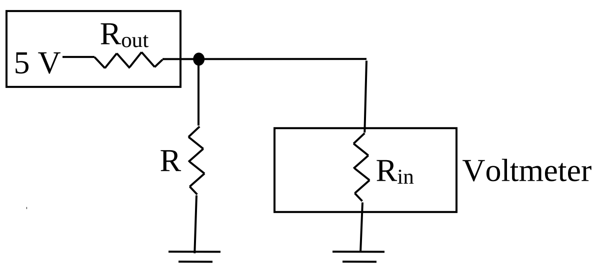

Figure 6. The output resistance of the voltage source is revealed.

There are three resistors in Figure 6. Let's first look at R and R_{in}. Let's assume that R << R_{in}. In this case, R dominates the parallel combination, and we can neglect R_{in}. Now we just have R_{out} in series with R. The current is then (5 V)/(R_{out} + R), and the voltage across R (which is measured by the voltmeter) is

$$V_{measured} = 5V \frac{R}{R_{out} + R}. \tag{2}$$

You can see that when R is much larger than R_{out}, $V_{measured}$ approaches 5 V; negligible voltage appears across R_{out}. However, when R becomes as small as R_{out}, $V_{measured}$ drops because some of the voltage appears across R_{out}. We don't want the voltage at the terminals of our voltage source to depend on R, so R_{out} should be as small possible;

smaller R_{out} values mean than R can be smaller before $V_{measured}$ is affected. **Low output resistance is good.**

Measuring Input Resistance

Suppose I have the following circuit:

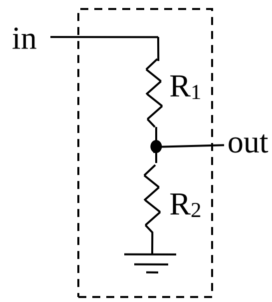

Figure 7. A circuit element known as a voltage divider.

Unlike the voltmeter in Figure 4 (which only receives input) or the voltage source in Figure 6 (which only supplies output), the circuit element in Figure 7 has both an input and an output terminal. Thus, this circuit element has both an input resistance and an output resistance.

You might want to know the input resistance of this circuit element (if only because that's your assignment in lab).

This is the way to measure the input resistance of an arbitrary circuit element:

1. Model the entire circuit as a single resistor R_{in} between the input and ground:

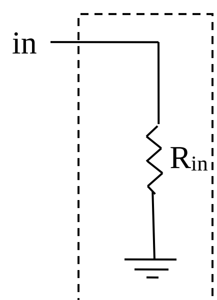

Figure 8. The model used to determine R_{in}.

So whatever your circuit it made of, whether it contains a hundred resistors or a thousand transistors, you model it as a single resistor R_{in} (just for this measurement).

2. Place a known resistor R in series with your circuit and apply a voltage V, as shown in Figure 9. (In this course, many circuit properties will depend on frequency, so we do NOT apply a dc voltage. Instead, we apply an ac voltage with a function generator. I refer to the amplitude of the ac signal as V.)

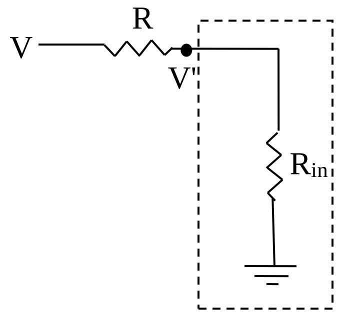

Figure 9. The circuit used to measure R_{in}.

3. Use an oscilloscope to measure the amplitudes V and V'. These two measurements can be done simultaneously or one at a time.

Knowing V, V', and R, you can determine R_{in}. The current through R is I = (V – V')/R. The same current flows through R_{in}: I = V'/R_{in}. We can combine these two equations to solve for R_{in}:

$$R_{in} = \frac{RV'}{V - V'} . \tag{3}$$

Measuring Output Resistance

We just saw how to measure the input resistance of the circuit element in Figure 7. We now want to know how to measure its output resistance. We follow the following steps:

1. Apply a voltage signal at the input of the circuit element in Figure 7. This voltage will not explicitly appear in the model we use to determine R_{out}.

2. Model the entire circuit as a voltage V in series with R_{out}. (This is called a Thevenin model, and sometimes R_{out} is called R_{Th}.) The voltage V in this model is the OUTPUT voltage for an "unloaded" circuit (a circuit with nothing connected between its output and ground).

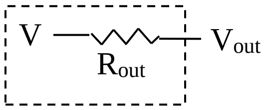

Figure 10. The model used to determine R_{out}.

3. Measure the amplitude V = V_{out} by connecting an oscilloscope to the output shown in Figure 10. (V = V_{out} because negligible current flows into the oscilloscope, so the current through R_{out} is zero, so the voltage across R_{out} is zero.)

4. Now connect a known resistor R between V_{out} and ground. Current now can flow through R_{out}, and V_{out} no longer equals V. Measure the new V_{out} (which I'll call V') with an oscilloscope. The measurements of V and V' cannot be done simultaneously!

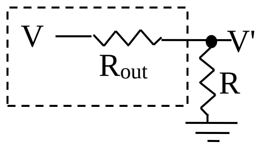

Figure 11. The "loaded" circuit (the load is the resistance between the output and ground).

It is extremely important to recognize that while the circuit is loaded (as shown in Figure 11), it is IMPOSSIBLE to measure V. You can only measure voltages that appear at the terminals of the dotted box. In Figure 11, V is "buried" inside the model, and only V' can be measured. Conversely, in Figure 10, only V (= V_{out} in this case) can be measured.

Knowing V, V', and R, you can determine R_{out}. Referring to Figure 11, the same current I flows through R_{out} and R. The current through R_{out} is $I = (V - V')/R_{out}$, and the current through R is $I = V'/R$. We combine these equations to solve for R_{out}:

$$R_{out} = R\frac{V - V'}{V'} . \qquad (4)$$